2nd year (Level 5) Physics courses

Handout
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Mathematical Methods and Applications

20 Credits

SUMMARY OF ... Handout 1

Semester 1
Mathematical Methods (formally called 'Theoretical Physics I')
Dr Graham S McDonald

Semester 2
Applications in Electricty, Magnetism and AC Circuit Theory
Dr Tiehan Shen

Assessment

Handout 1
pages 1 & 2

January Test 40%

May Exam 60%

Core Books

Semester 1 - K.A Stroud, 'Advanced Engineering Mathematics' 4th Ed, Macmillan Press 2003

Semester 2 - University Physics with Modern Physics, by Young and Freedman 13th Edition (2011), Pearson

Syllabus outline

Handout 1
pages 1 & 2

<u>Semester 1 – Dr Graham S McDonald</u>

- Vector calculus, including: gradient, divergence, flux and curl, the divergence theorem and Stokes' theorem.

- Matrices, determinants, eigenvalues and eigenvectors. Applications of matrices.

- Partial differential equations and methods of solution, e.g. separation of variables.

<u>Semester 2 – Dr Tiehan Shen</u>

The magnetic field. Biot and Savart law and Ampere's law.
Electromagnetic induction. Magnetic flux; Faraday's and Lenz's law
Transients in LR, RC and LCR circuits;
AC Theory and complex analysis: reactance, impedance and resonance

REVIEW OF FUNDAMENTAL CONCEPTS (part one)

This section includes revision of first year material ...

H1
p5,6

Scalar versus vector quantities
Magnitude of a vector, and a unit vector
Physical examples
Equality of vectors
Components of vectors (Cartesian co-ordinates)
Magnitude (in terms of components)
Addition of vectors
Multiplication by a scalar
Difference of vectors
i , **j** , **k** unit vectors

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and ...

DOT PRODUCT OF VECTORS

The dot product of vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is

$$\boxed{\underset{\sim}{a}\cdot\underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta},$$

where θ is the angle between the vectors.

Everything on the RHS, i.e. $|\underset{\sim}{a}|$, $|\underset{\sim}{b}|$ and $\cos\theta$, are just numbers (scalars). So the dot product of two vectors is just a number. It is also called the SCALAR PRODUCT.

a.b is:
positive, zero and negative ...
for angles:
acute, 90 degrees, obtuse (respectively).

If $\underset{\sim}{a} = (a_1, a_2, a_3) = a_1\underset{\sim}{i} + a_2\underset{\sim}{j} + a_3\underset{\sim}{k}$

and $\underset{\sim}{b} = (b_1, b_2, b_3) = b_1\underset{\sim}{i} + b_2\underset{\sim}{j} + b_3\underset{\sim}{k}$,

$$\boxed{\underset{\sim}{a}\cdot\underset{\sim}{b} = a_1b_1 + a_2b_2 + a_3b_3}$$

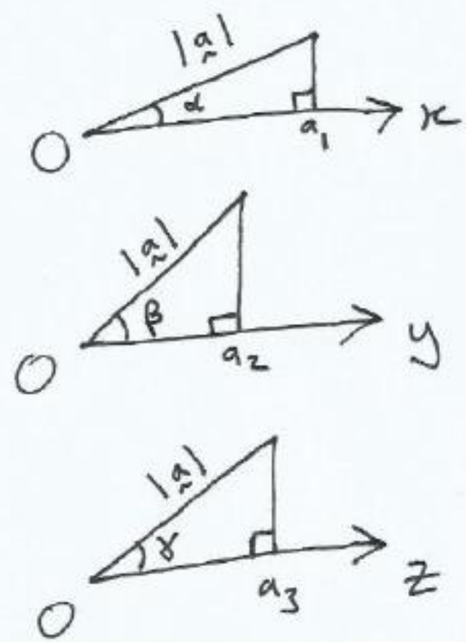
Component form

p14 to p16

and something new ...

Specifying direction of a vector (length-independent):

'DIRECTION COSINES' appear in solid-state theory and express angles made with each Cartesian axis... for $\underset{\sim}{a} = (a_1, a_2, a_3)$,



$\cos\alpha = l$

$\cos\beta = m$

$\cos\gamma = n$

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Using the dot product to find the angle between two vectors is **formally equivalent** to working out the direction cosines $[l, m, n]$ and $[l', m', n']$ of $\underset{\sim}{a}$ and $\underset{\sim}{b}$, respectively, and using the result that

$$\boxed{\cos\theta = ll' + mm' + nn'}$$

Since $l = \frac{a_1}{|\underset{\sim}{a}|}$, $m = \frac{a_2}{|\underset{\sim}{a}|}$, $n = \frac{a_3}{|\underset{\sim}{a}|}$

and $l' = \frac{b_1}{|\underset{\sim}{b}|}$, $m' = \frac{b_2}{|\underset{\sim}{b}|}$, $n' = \frac{b_3}{|\underset{\sim}{b}|}$, This brings us to the notion of a FIELD.

A field is basically a region of space where quantities, such as force, may assume different values depending upon where one is within this space.

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More technically, a 'field' is actually the physical quantity itself and the 'space' can include time.

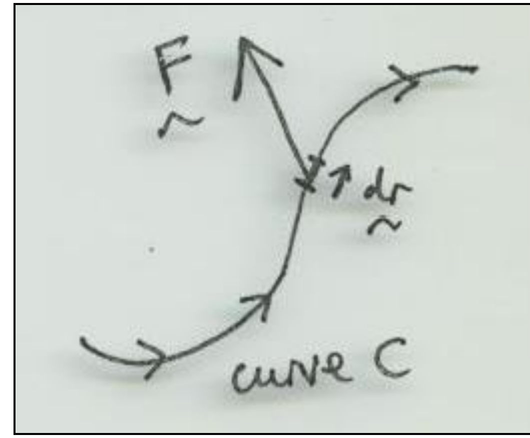
There are two types of field :

- scalar field – where a scalar quantity can have different values in different places. If the scalar quantity is denoted ϕ then $\phi(x,y,z)$.

- vector field – where a vector quantity may have different values in different places. If the vector quantity is denoted V then $\underset{\sim}{V}(x,y,z)$.

Some physical examples (e.g. in a weather map) are then given.

Calculating work done by a spatially-varying force *F* (*x,y,z*) acting along a general curve, C



Work done over ***dr = F . dr***

Work done along C , **W =** $\int_C \boldsymbol{F}.\,\boldsymbol{dr}$

... a "line integral"

In component form,

$$\mathbf{W} = \int_C (\boldsymbol{F_x i} + \boldsymbol{F_y j} + \boldsymbol{F_z k}).(\boldsymbol{dx i} + \boldsymbol{dy j} + \boldsymbol{dz k})$$

$$= \int_c \boldsymbol{F_x dx} + \boldsymbol{F_y dy} + \boldsymbol{F_z dz}$$

Then **2 examples** (we will go over the details in handout 2). Firstly ...

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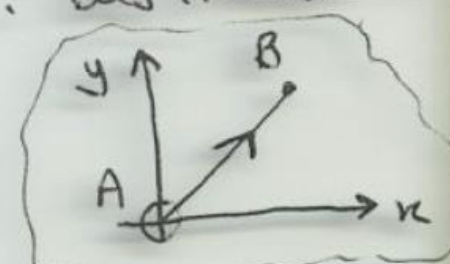
$\underset{\sim}{F}(x,y,z)$ defines a vector field, but are there different types of vector field?

Ex Particle moving in the x-y plane under the influence of force $\boxed{\underset{\sim}{F} = (y^2, x^2, 0)}$. What is the work done in going from A at $(x,y) = (0,0)$ to B at $(x,y) = (1,1)$?

Ans What route do we take? Where is curve C? Does it matter?

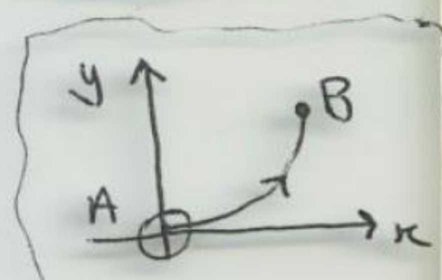
Let's try two different routes (a) along $y = x$



and secondly, the same but with ...

$\boxed{\underset{\sim}{F} = (xy^2, yx^2, 0)}$

(b) along $y = x^2$



Two examples (two different types of force) ...

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The first example gave different work done along different paths – a "**non-conservative force**" (also called a **dissipative** force/field).

The second example gave the same work done along the different paths. If this is true for *all paths*: a "**conservative force".**

<u>Conservative case</u>. In other words, the *work done only depends on the start and end points* (A and B, respectively).

So, we could write ...

$$\int_A^B \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_A^B dW = W_B - W_A$$

where W(x,y,z) is a scalar function of space, and $W_B - W_A$ is the difference in values of W (concerning end/start points).

p29-p30

$$\int_A^B \underset{\sim}{F} \cdot d\underset{\sim}{r} = \int_A^B dW = W_B - W_A$$

(i) **PATH INDEPENDENCE**

Since $\underset{\sim}{F} \cdot d\underset{\sim}{r} = F_x dx + F_y dy + F_z dz$ then this is equivalent to

$$\boxed{F_x dx + F_y dy + F_z dz = dW = \text{PERFECT/EXACT DIFFERENTIAL}}$$

(ii)

(iii)

One can then show (details in handout 2) that, for **F** to be conservative, its components need to satisfy ...

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

$$\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}$$

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$$

This gives us a means to **test** whether a given **F** is conservative.

But **(i)**, **(ii)** and **(iii)** are just properties of a field - not just a force field ...

 (details in handout 2).

Are there general classes of conservative field (not just force fields)? What about:

$$\boxed{\boldsymbol{V}(\boldsymbol{r}) = \frac{\eta\,\hat{\boldsymbol{r}}}{r^2}}$$

: <u>*any*</u> field obeying a **radial, inverse-square** law ?

where η is a physical constant

r is distance from an origin

$\hat{\boldsymbol{r}}$ is a (radial) position vector = $\frac{\boldsymbol{r}}{r}$

If we apply the test, labelled **(iii)** earlier, ... **YES !**