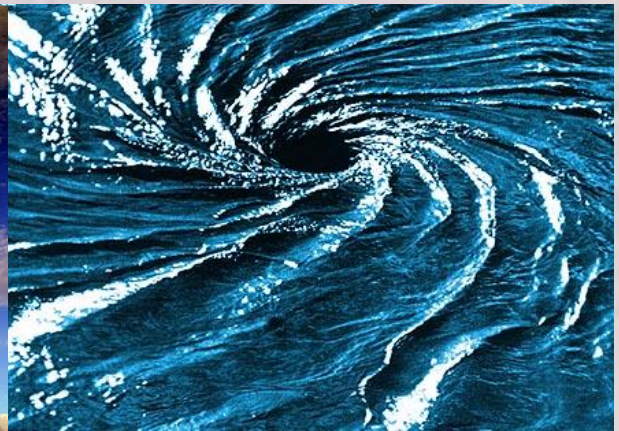
University of Salford
Institute for Materials Research

COMPLEXITY, THE PHYSICS OF EVERYTHING

Dr Graham S McDonald
Joule Physics Lab
University of Salford



< Event information >

“Complexity” - the physics of everything?

Complexity **is a subject that bridges across diverse fields**

Common aspects **of these fields are being identified as due to the *same processes***

Knowledge and techniques from each field can then be used in the other fields …

Outline

- Simple patterns vs. Complex patterns
- Feedback
- Perspective on fractals
- Origin of simple patterns
- Applications
  - photonics
  - an origin of fractal patterns
  - global human population

Simple patterns vs. Complex (fractal) patterns

Results from many very *complicated systems* are actually determined by *similar simple mechanisms*

These simple mechanisms tend to result in:

1. SIMPLE PATTERNS

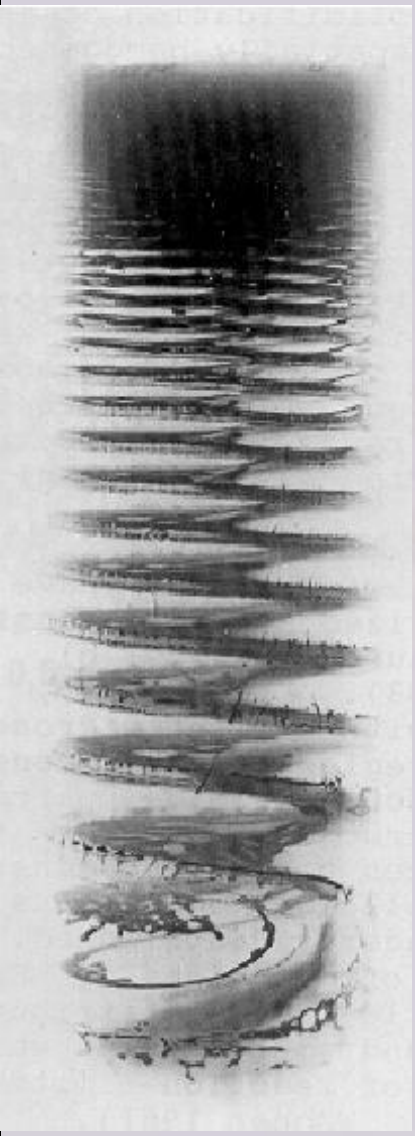
OR

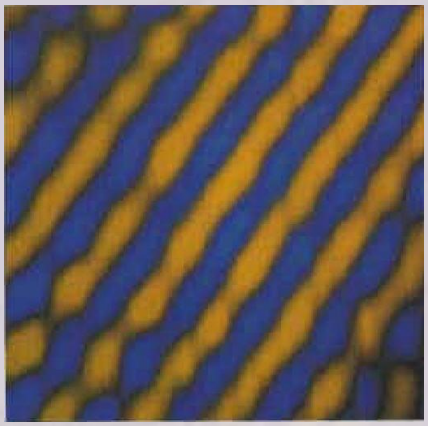
2. VERY COMPLEX PATTERNS (“fractals”)

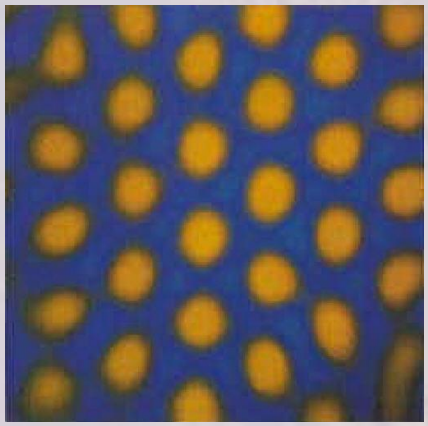
1. SIMPLE PATTERNS

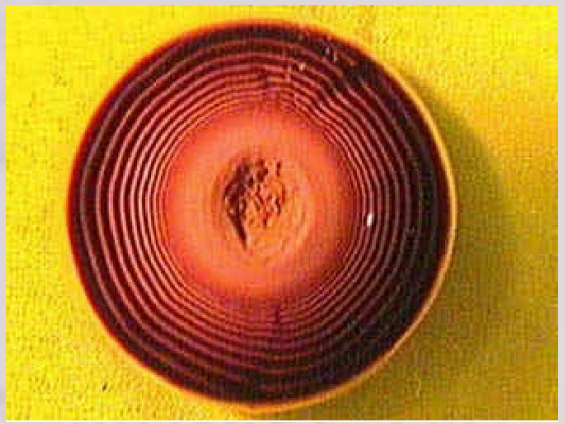
(e.g single spots,
vortices,
spirals,
honeycombs,
stripes, …)

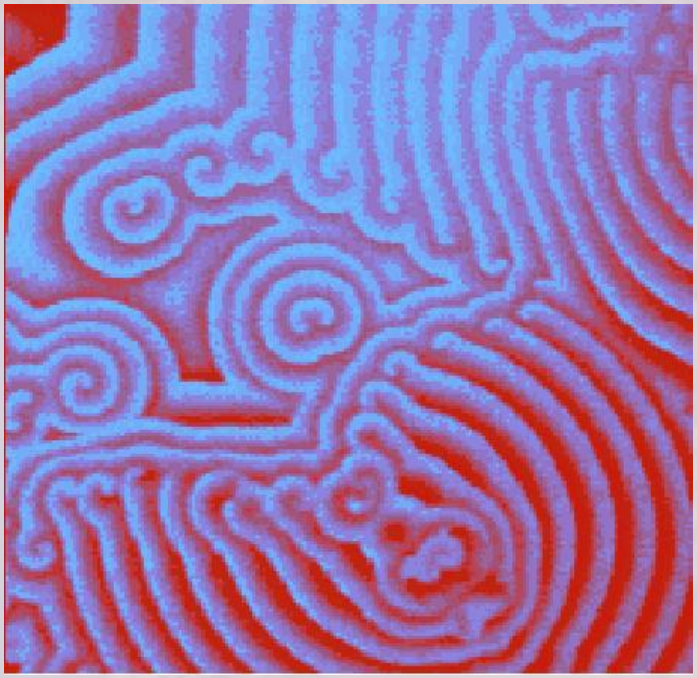
Spontaneous patterns in chemical reactions

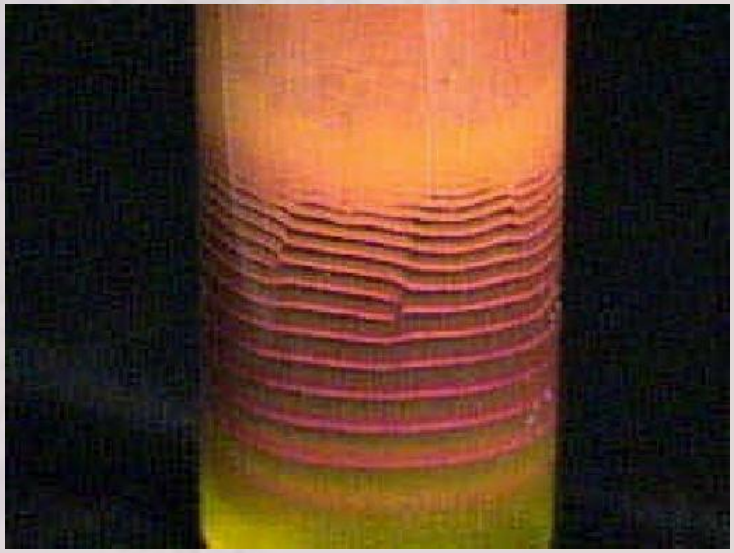




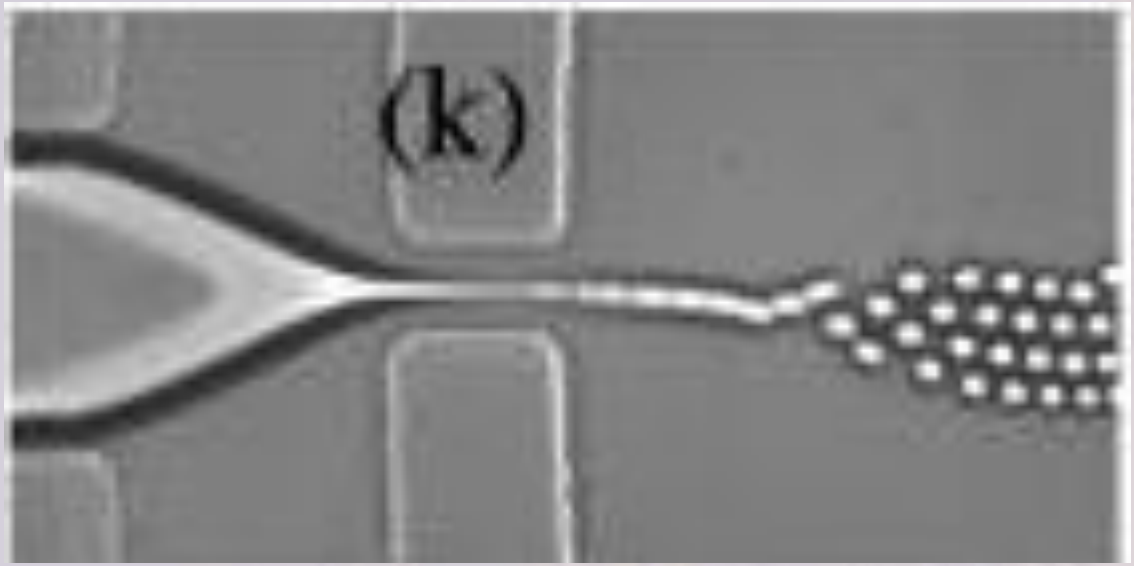








(A Goryachev, A'Star, Singapore)



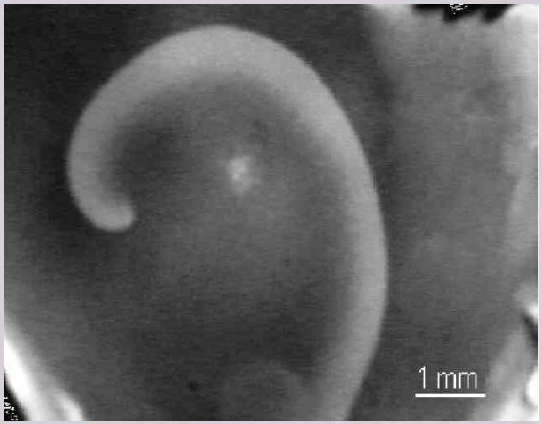
PATTERN OF
WATER DROPLETS

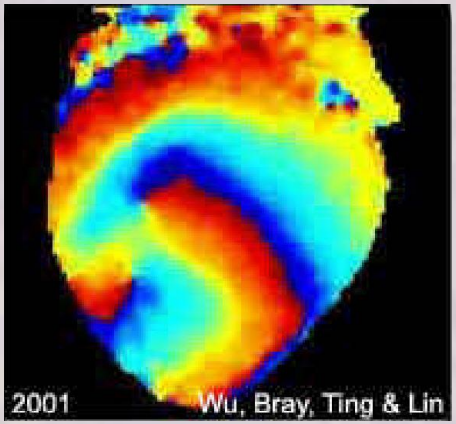
→

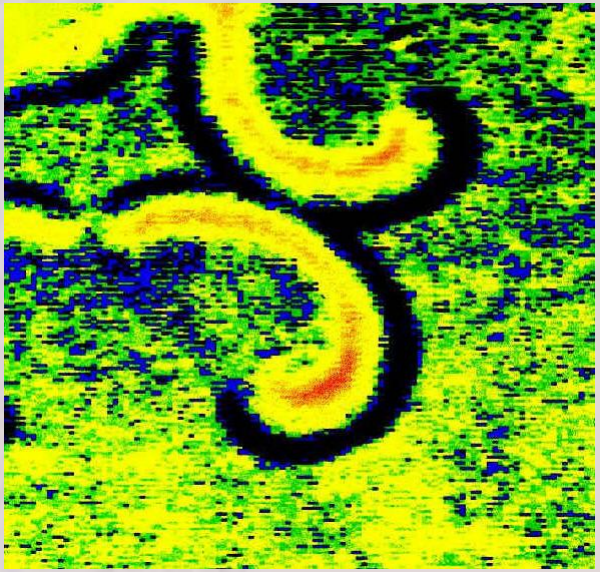
NARROW JET OF WATER
PASSING THROUGH A GAP IN OIL

a spontaneous hexagonal pattern
of water droplets forms …

Spiral waves in biological systems

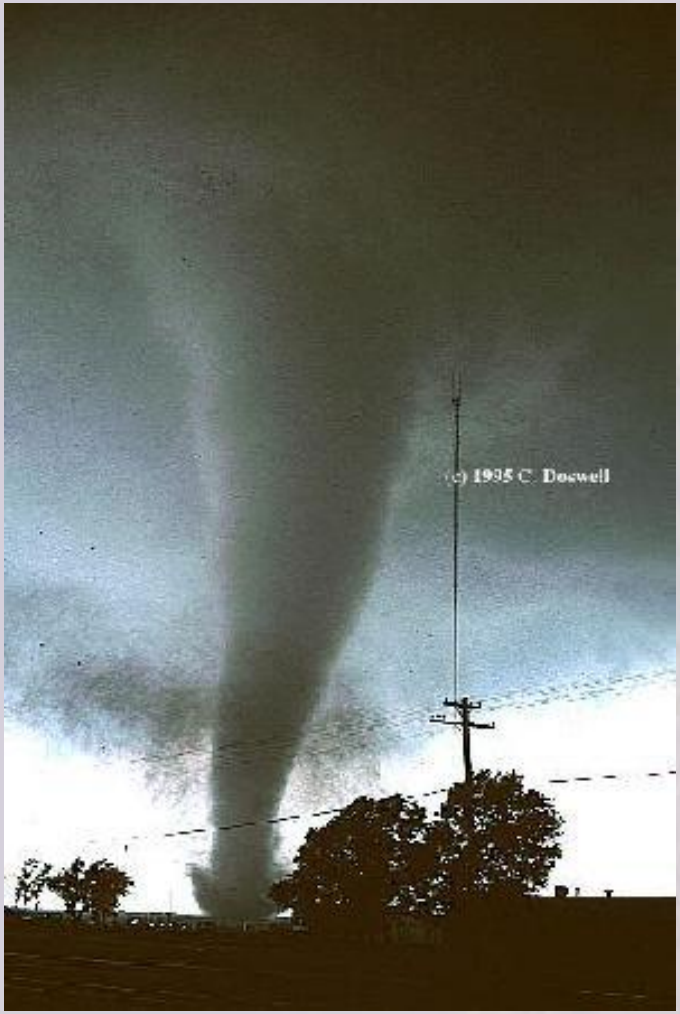


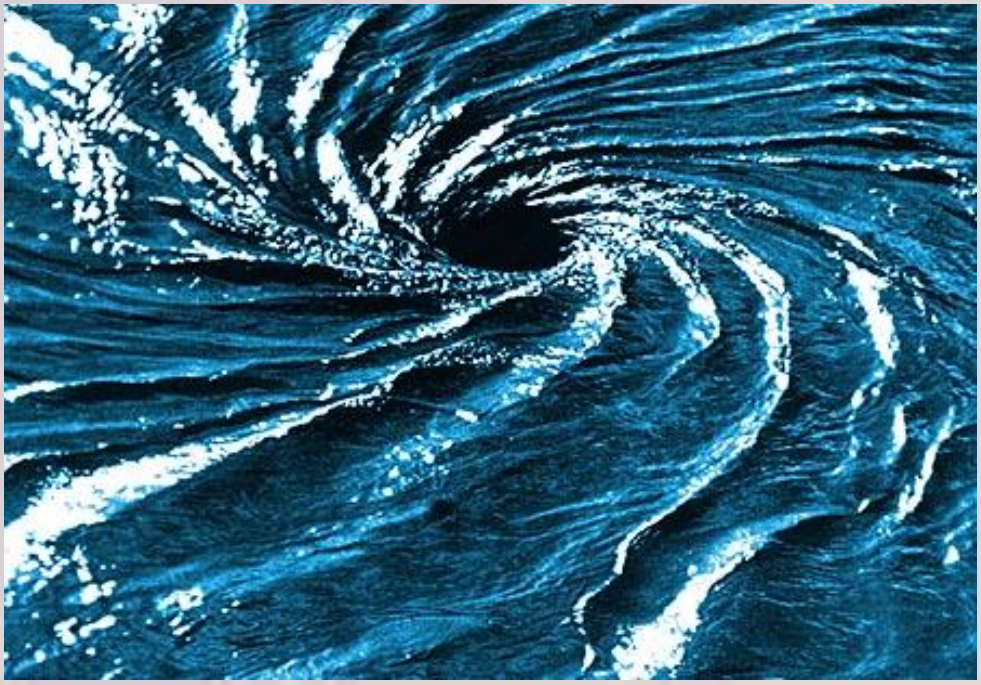




(A Goryachev, A'Star, Singapore)

“Dark spots”

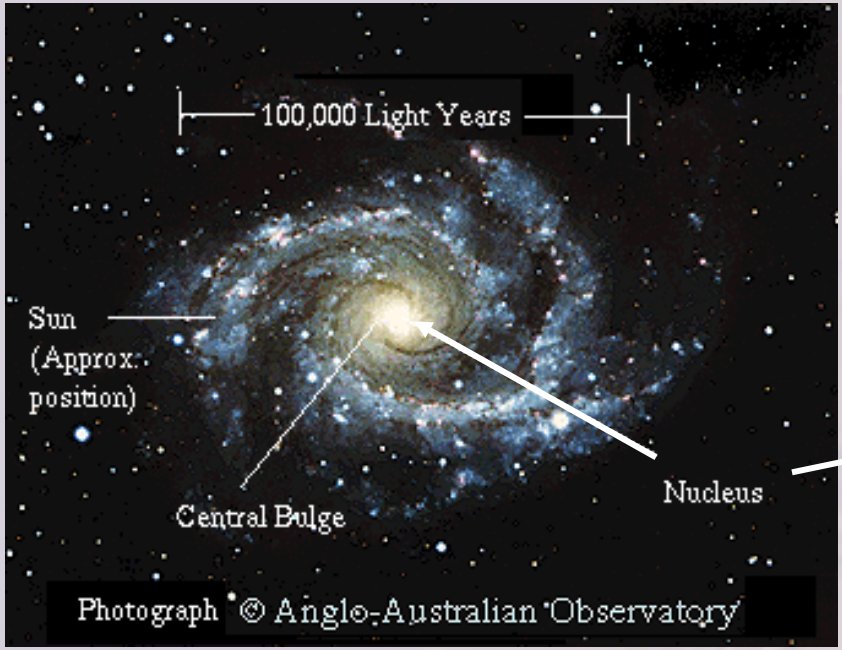


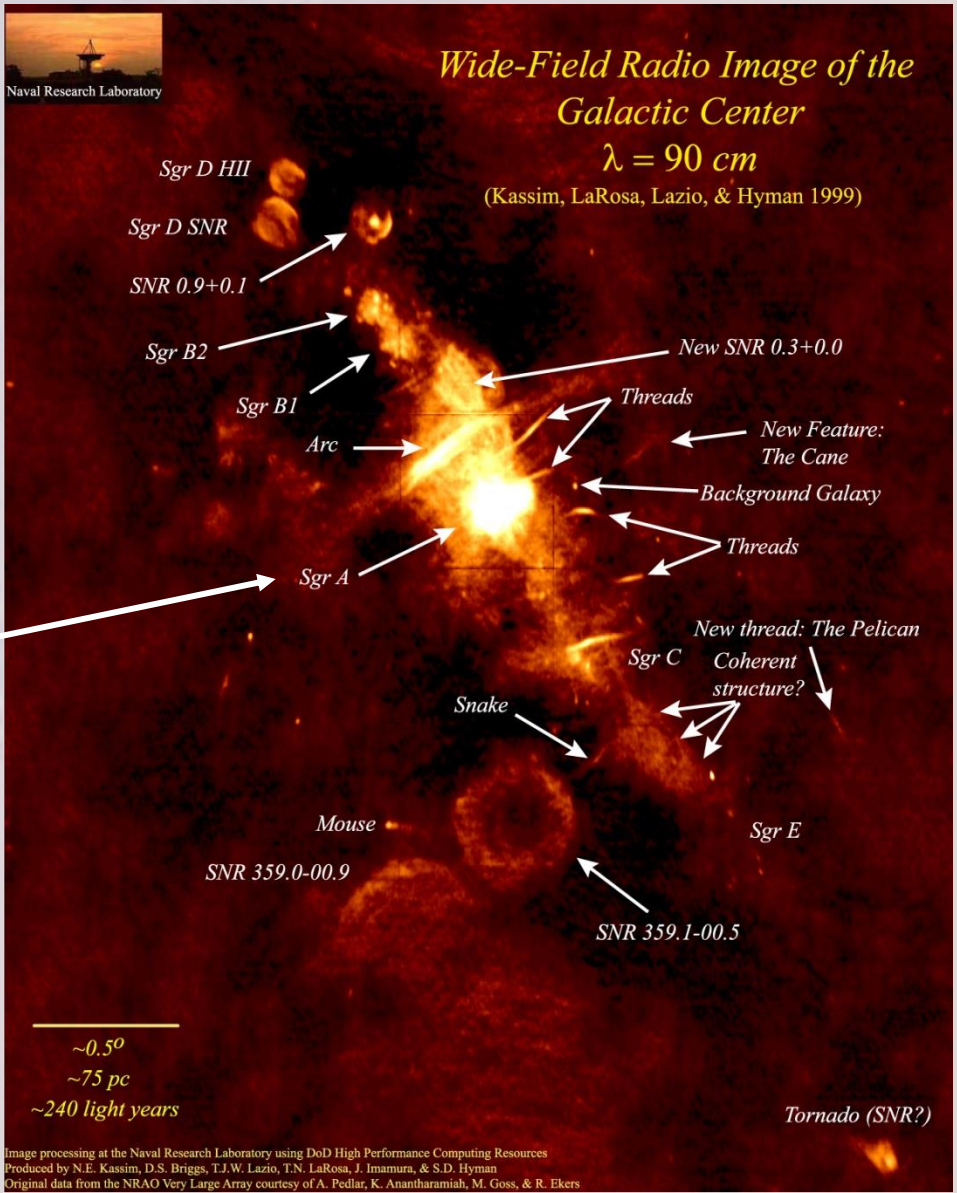


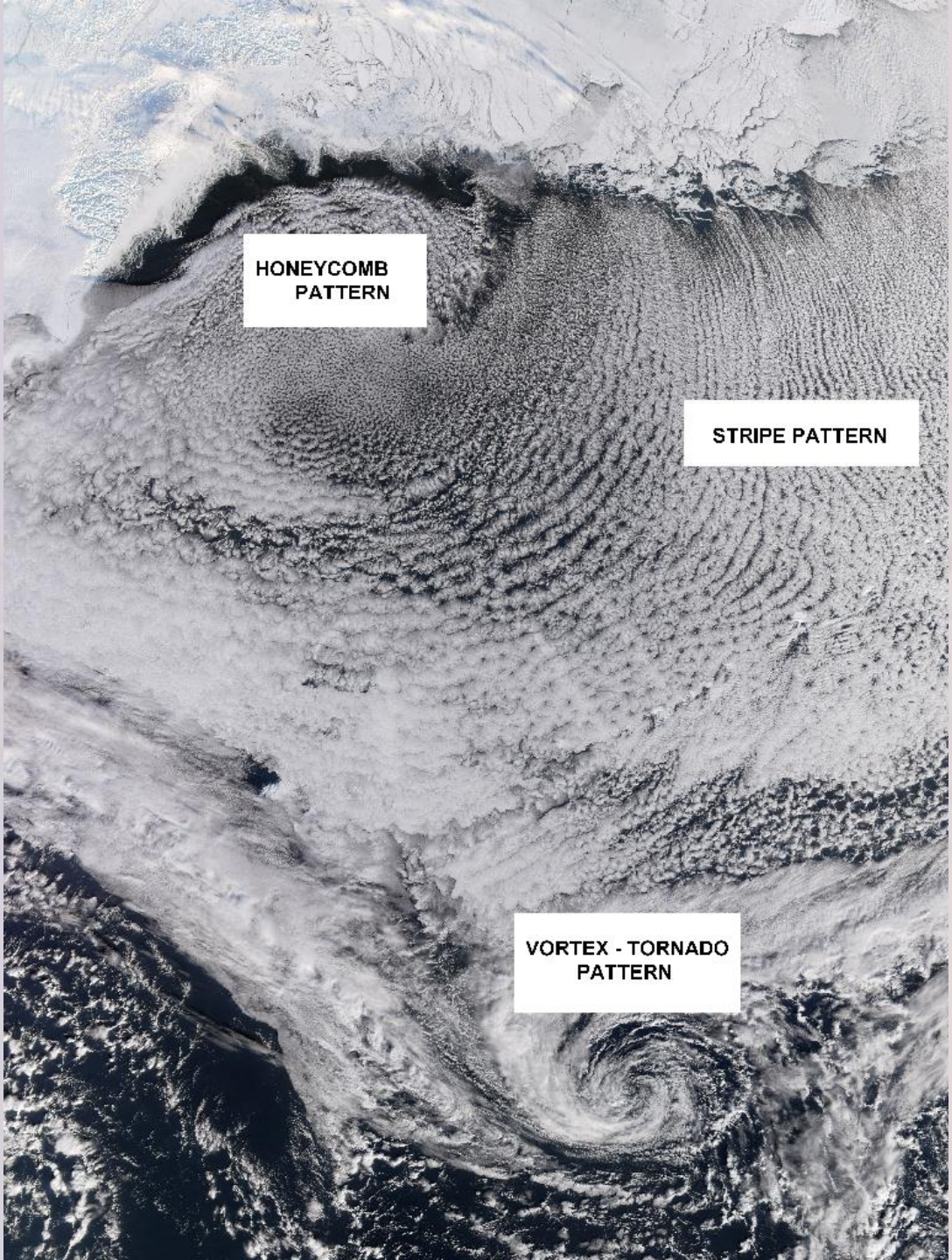
FLUID VORTEX

ATMOSPHERIC VORTEX
(TORNADO)

… relatively new “dark spots”






HONEYCOMB
PATTERN
STRIPE PATTERN
VORTEX - TORNADO
PATTERN

2. VERY COMPLEX PATTERNS

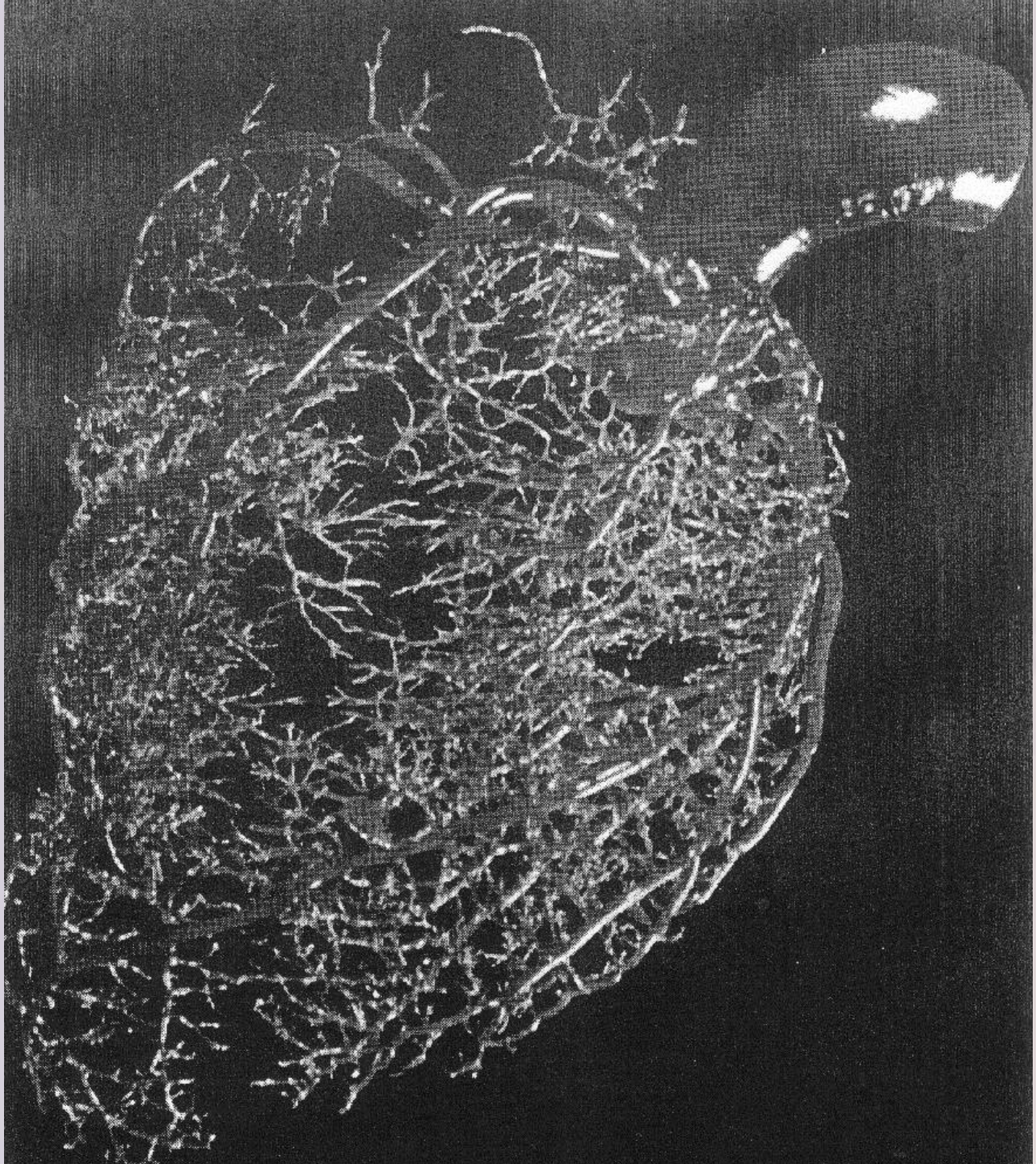
(“*fractals*”:

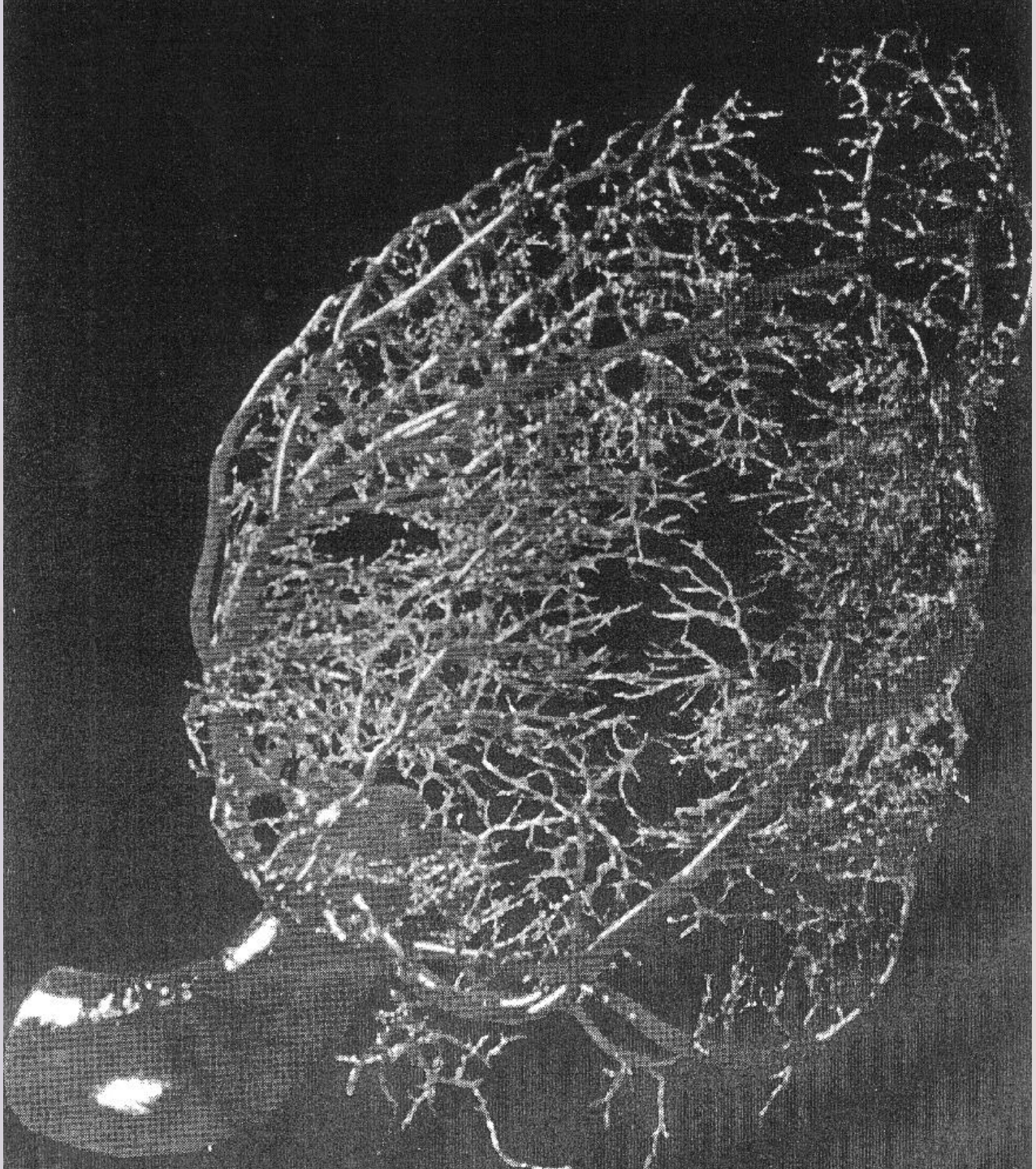
patterns with

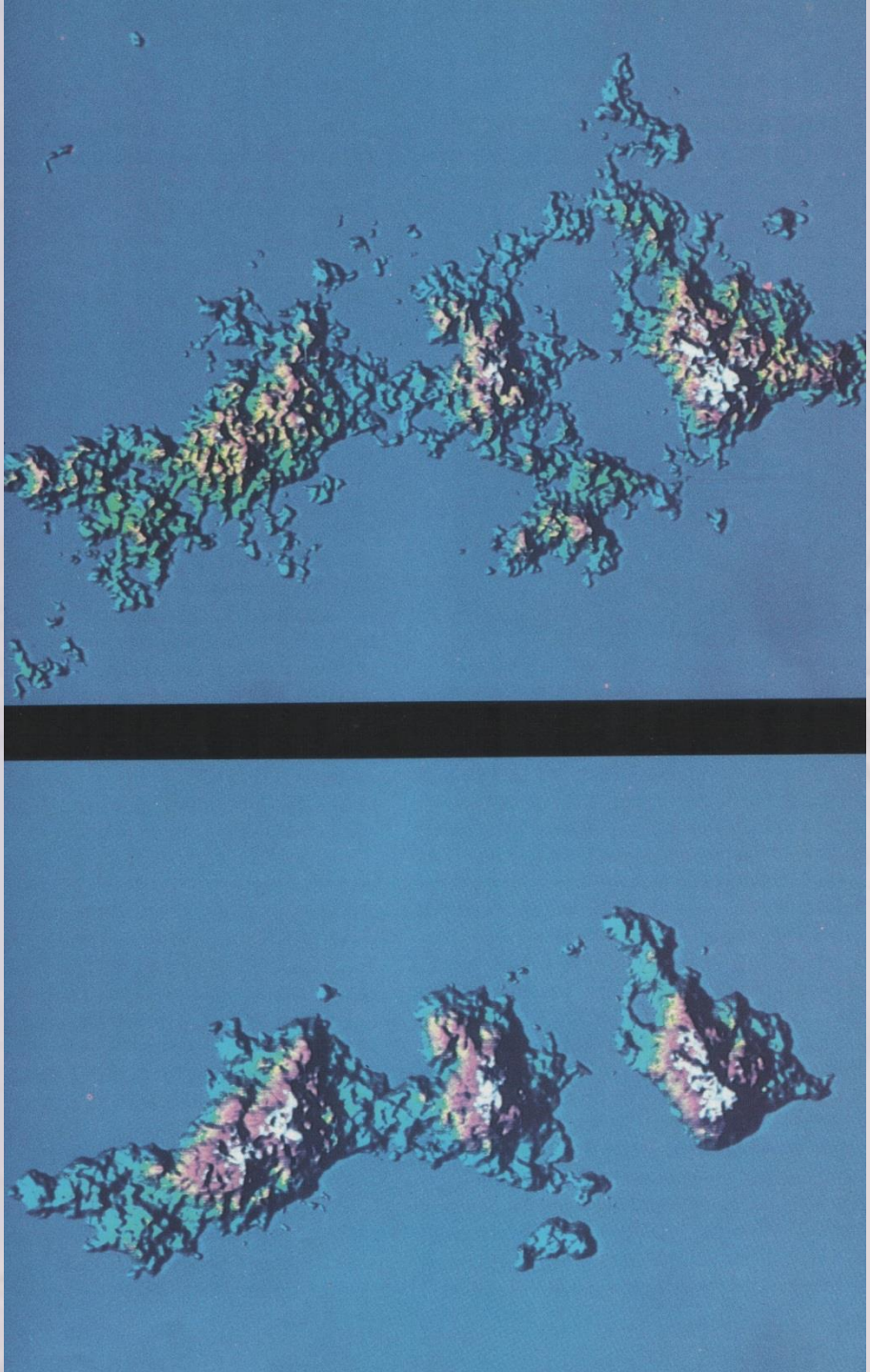
more & more detail

evident when you

look closer & closer)

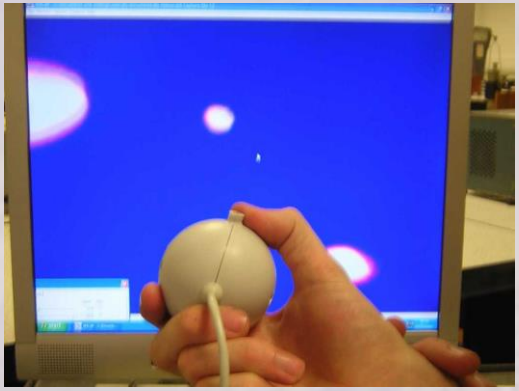


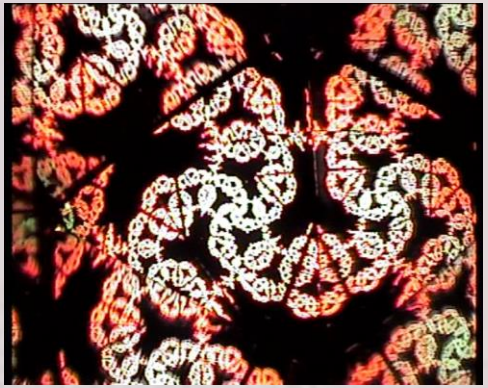




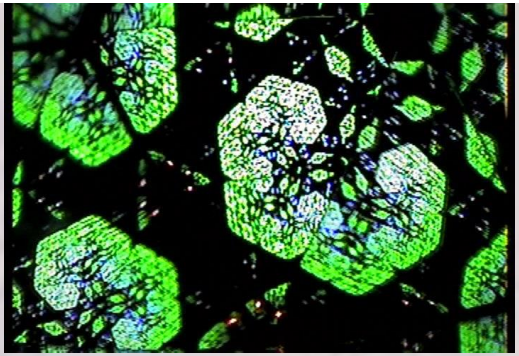
next …

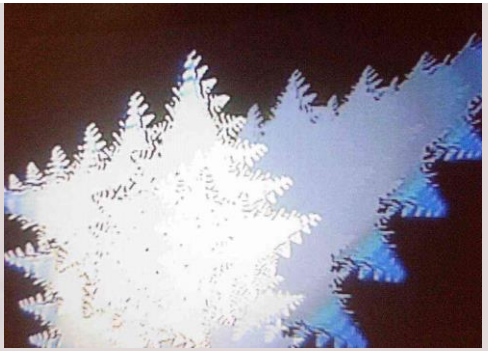
- Simple patterns vs. Complex patterns
- Feedback …





VIDEO FEEDBACK AT SALFORD UNIVERSITY



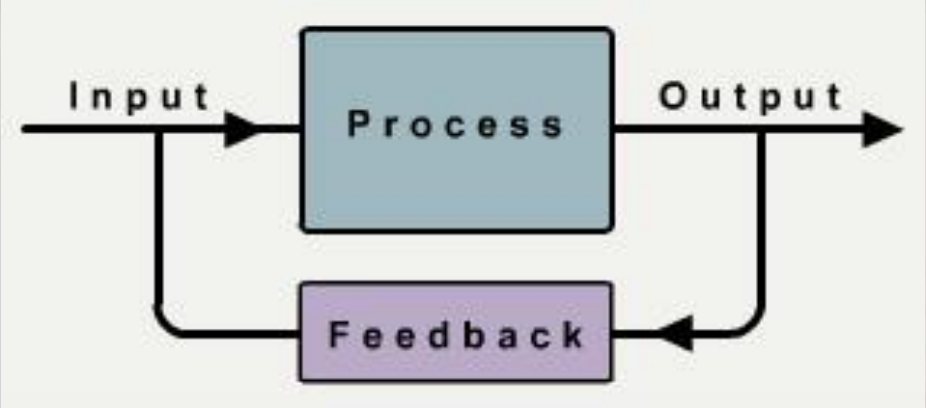


- Perspective on fractals
- Origin of simple patterns
- Applications
  - photonics
  - an origin of fractal patterns
  - global human population

The ***SIMILAR SIMPLE MECHANISMS***

at work in these complex systems is

FEEDBACK …



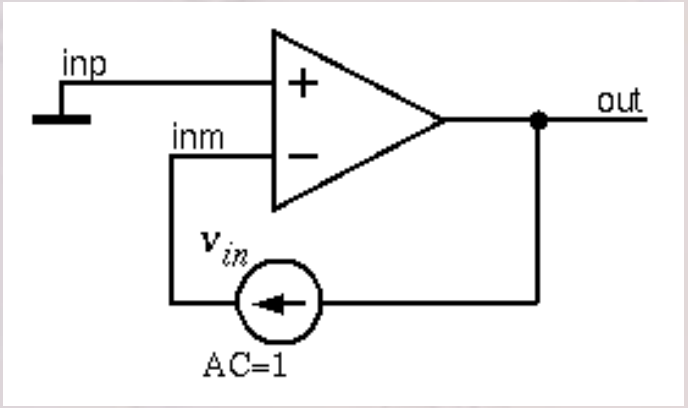
Examples of feedback

- Audio feedback

AUDIO OUTPUT INPUT (MICROPHONE)

- Electronic feedback

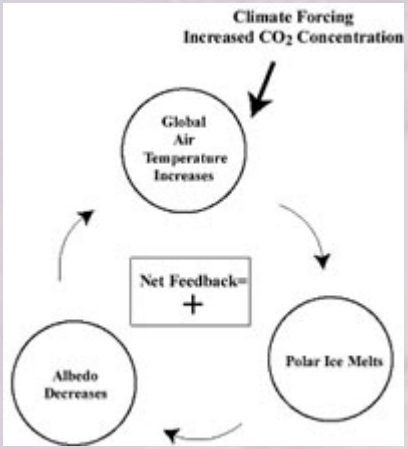
INPUT SIGNAL



OUTPUT SIGNAL

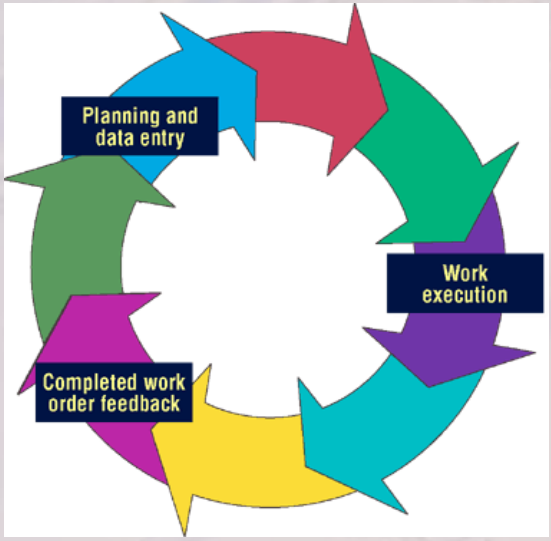
… more examples of feedback

- Weather systems



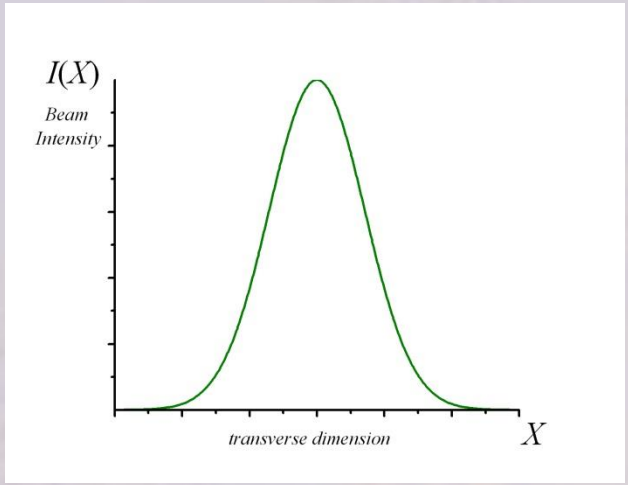
scenarios for global warming

- Management, business & manufacturing systems



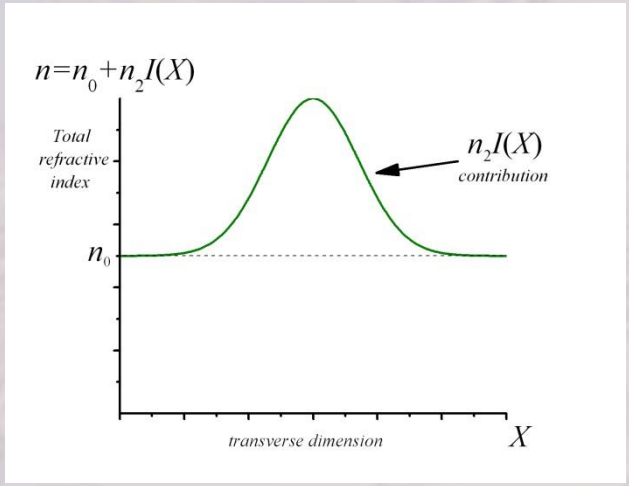
… more examples of feedback

- “Kerr effect” (eg optical systems)



Cross-sectional beam profile

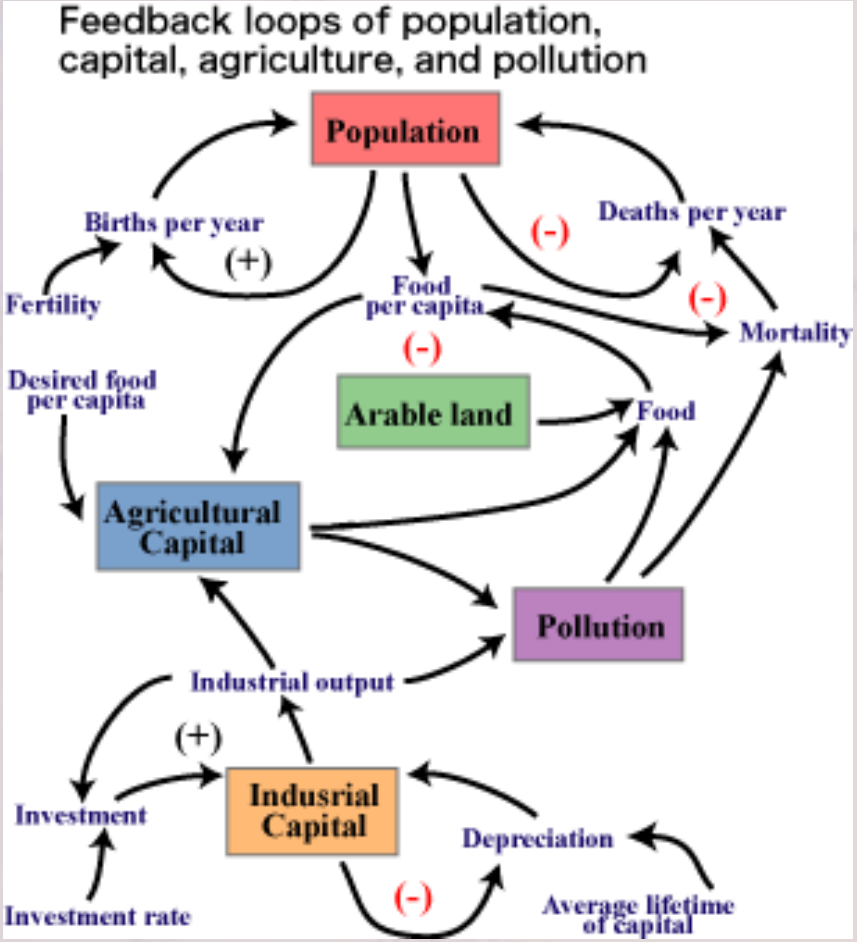
Induced lens focuses beam profile



Induces lens *n*(*I*) in medium

Individual systems complicated and may be coupled to other complex systems …

… but
a SINGLE, DOMINANT FEEDBACK LOOP
tends to prevail



A dominant feedback loop gives rise to the emergence and persistence of one of two extremes:

1. SIMPLE PATTERNS

e.g. spots (bright & dark), or stripes,
or hexagon patterns
– characterised by *a single scale*

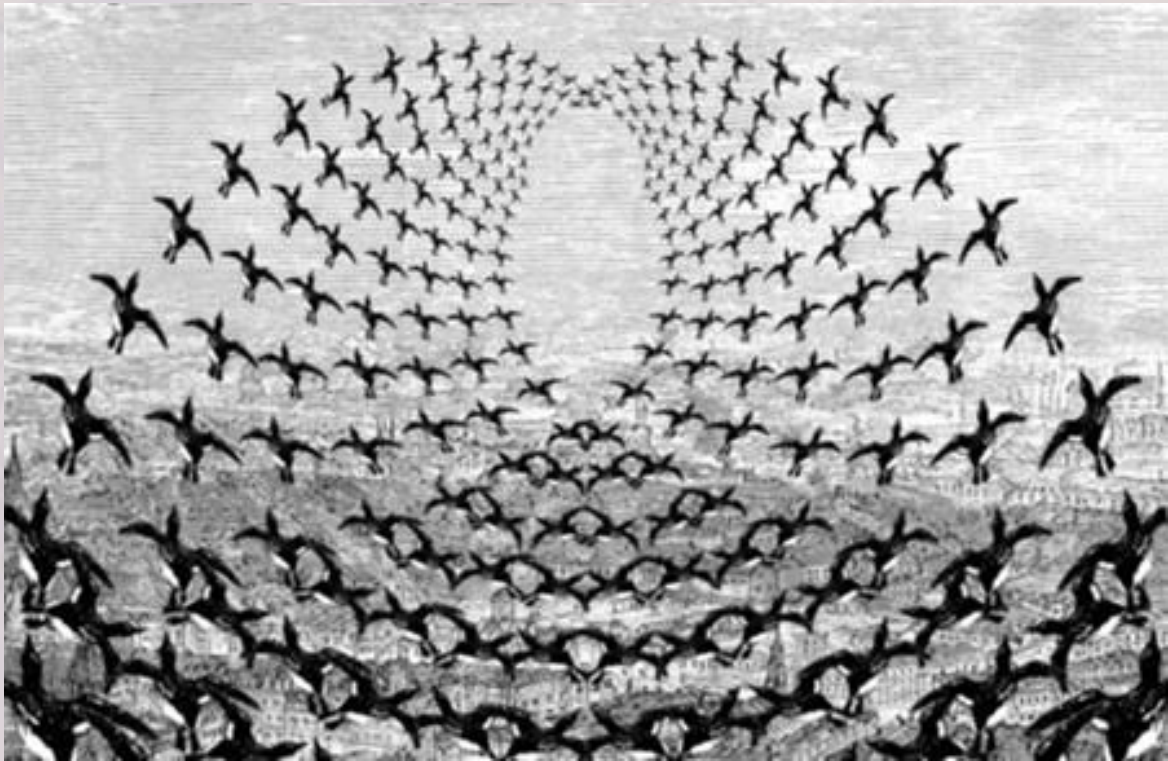
OR

2. VERY COMPLEX PATTERNS

"*fractals*": patterns with more & more detail
when looking closer & closer
- with structure on *all scales*

The nature of the feedback loop characterises the outcomes … rather than details of the physical systems

BIRDS FLOCKING



EXAGGERATED FLOCKING BEHAVIOUR !

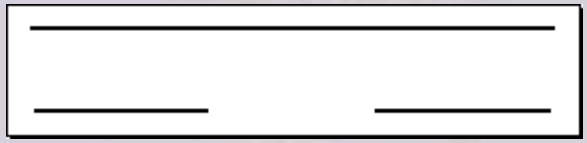
next …

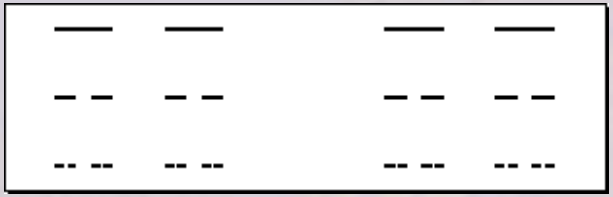
- Simple patterns vs. Complex patterns
- Feedback
- Perspective on fractals …
- Origin of simple patterns
- Applications
  - photonics
  - an origin of fractal patterns
  - global human population

“Exact fractals”

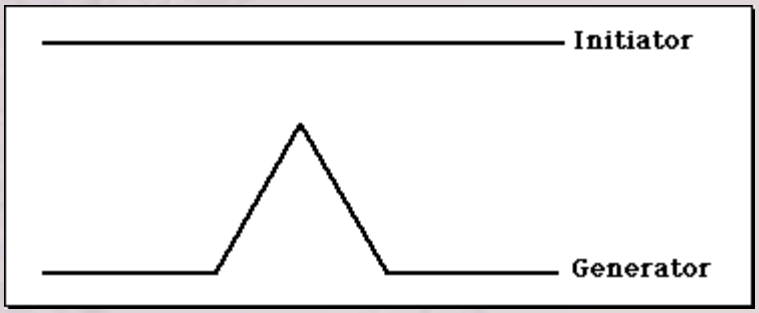
Initiator

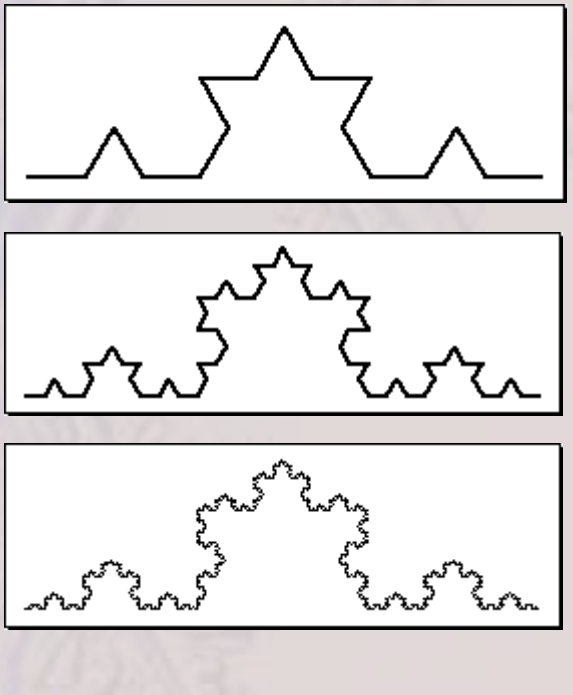
Generator





Cantor set





Koch snowflake

Georg Cantor
(born 1845)

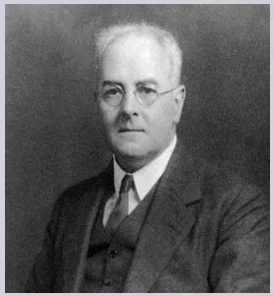
Neils Koch
(born 1870)

Exact scale-less patterns in nature?

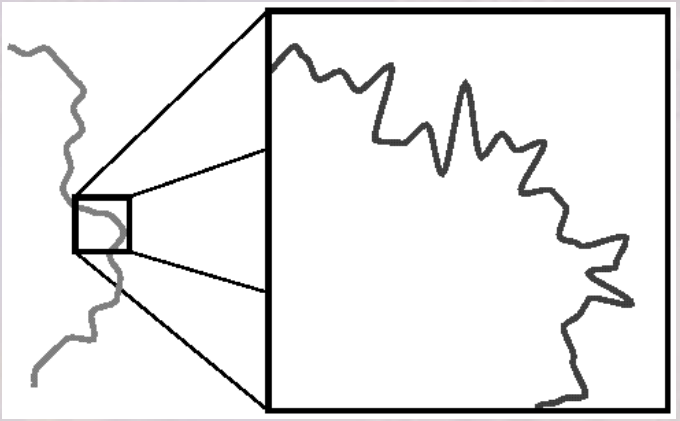
Idealised representations

- Not exact but *statistical*
- Structure on many (*but not all*) scales

Length of a coastline



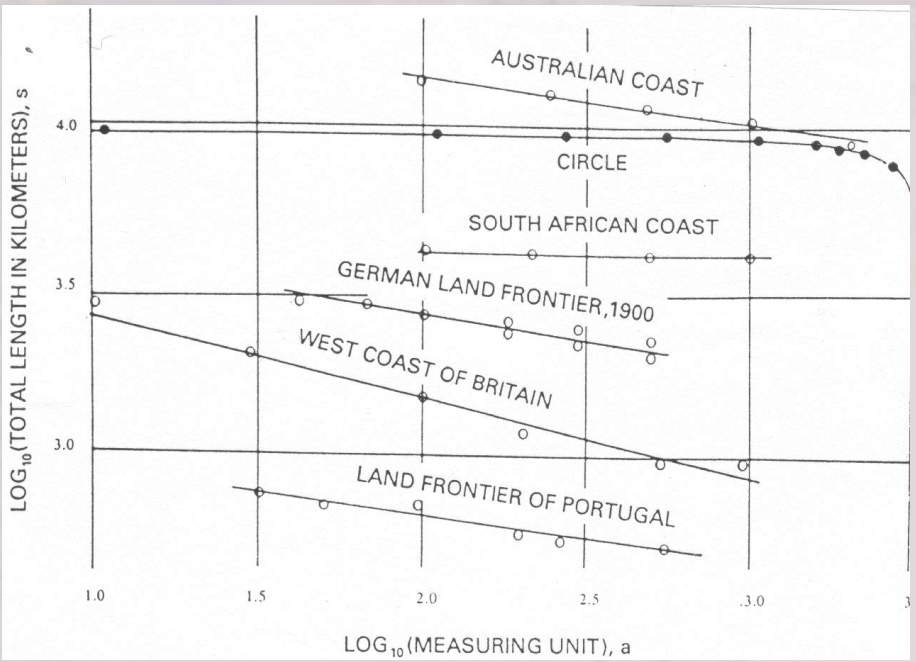
Lewis Fry Richardson
(born 1881)



Total length *L* depends on *map scale*

$\log_{10}(L)$

for *ruler length d*



$\log_{10}(d)$

Number of rulers $N(d)$ for total length L

d = 200 km *d* = 100 km *d* = 50 km

(*d* = ruler size)

Non-fractal:

$$d \times N(d) = L$$

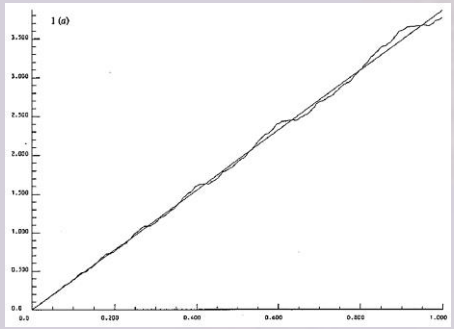
Coastlines:

$$d^D \times N(d) = L$$

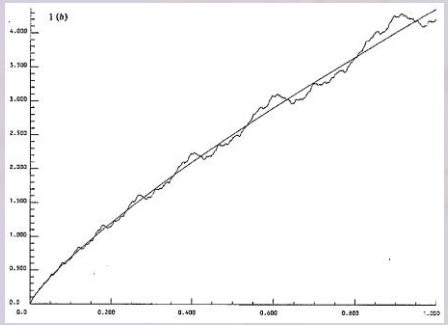
The “Capacity Dimension”
(capacity to fill space)

$$D = \frac{\log\left[N(d)\right]}{\log\left[1/d\right]}$$

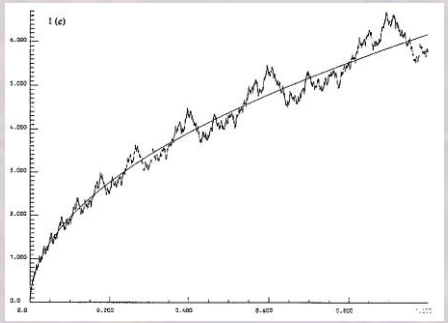
Notion of “roughness”



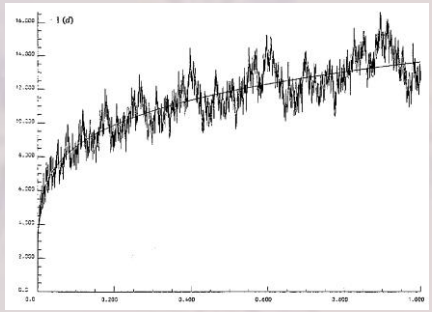
D = 1.0 (non-fractal)



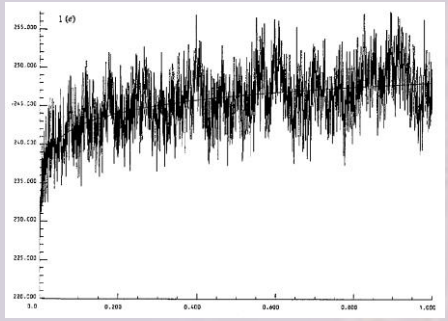
D =1.2



D =1.5



D =1.8



D =1.99

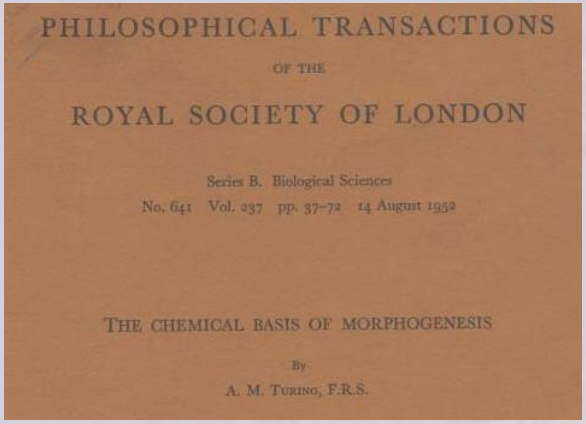
From *On the Weierstrass-Mandelbrot fractal function*
by MV Berry & ZV Lewis,
Proc R Soc Lond A 370, 459 (1980)

next …

- Simple patterns vs. Complex patterns
- Feedback
- Perspective on fractals
- Origin of simple patterns …

- Applications
  - photonics
  - an origin of fractal patterns
  - global human population

Turing instability

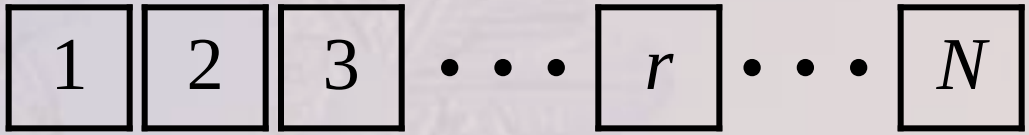

PHILOSOPHICAL TRANSACTIONS
OF THE
ROYAL SOCIETY OF LONDON

Series B. Biological Sciences
No. 641 Vol. 237 pp. 37–72 14 August 1952

THE CHEMICAL BASIS OF MORPHOGENESIS

By
A. M. TURING, F.R.S.

Alan Turing
(1912-1954)



N cells, with species concentrations X_r and Y_r

$$\frac{dX_r}{dt} = f(X_r, Y_r) + \mu(X_{r+1} - 2X_r + X_{r-1})$$

$$\frac{dY_r}{dt} = g(X_r, Y_r) + \nu(Y_{r+1} - 2Y_r + Y_{r-1})$$

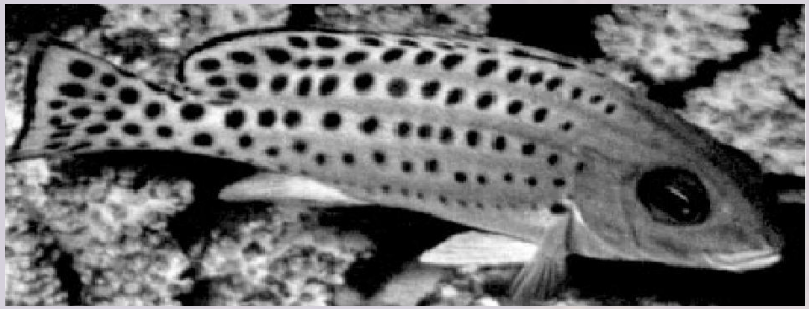
reaction diffusion

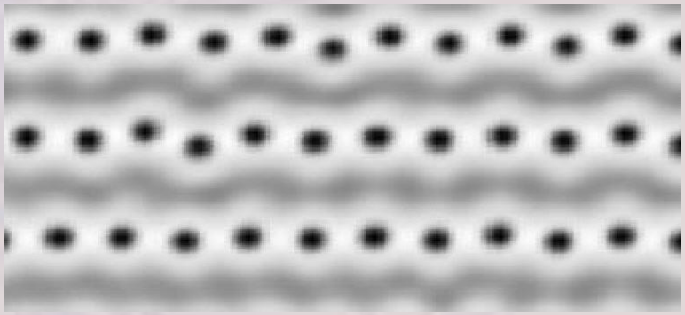
- Small disturbances → birth of simple patterns
- Universal applications

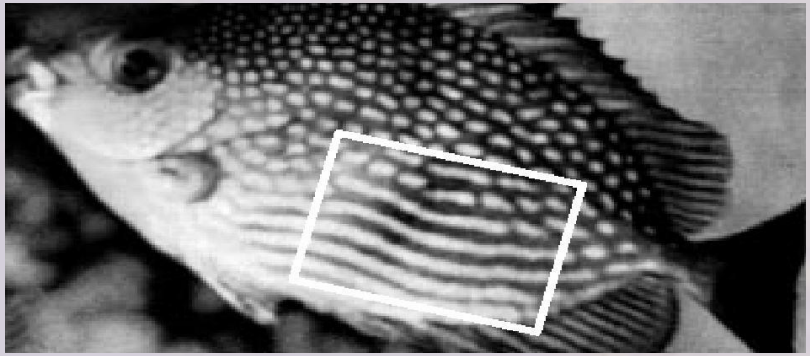
Fish skins & simulated patterns

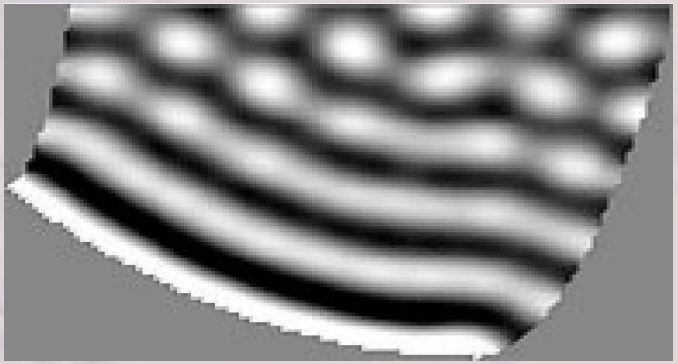
FISH

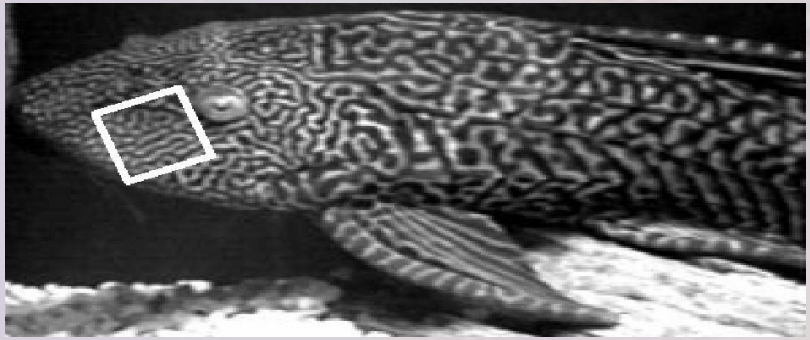
SIMULATED PATTERNS

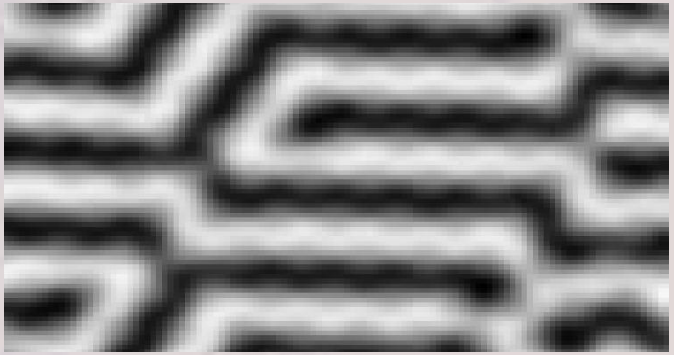






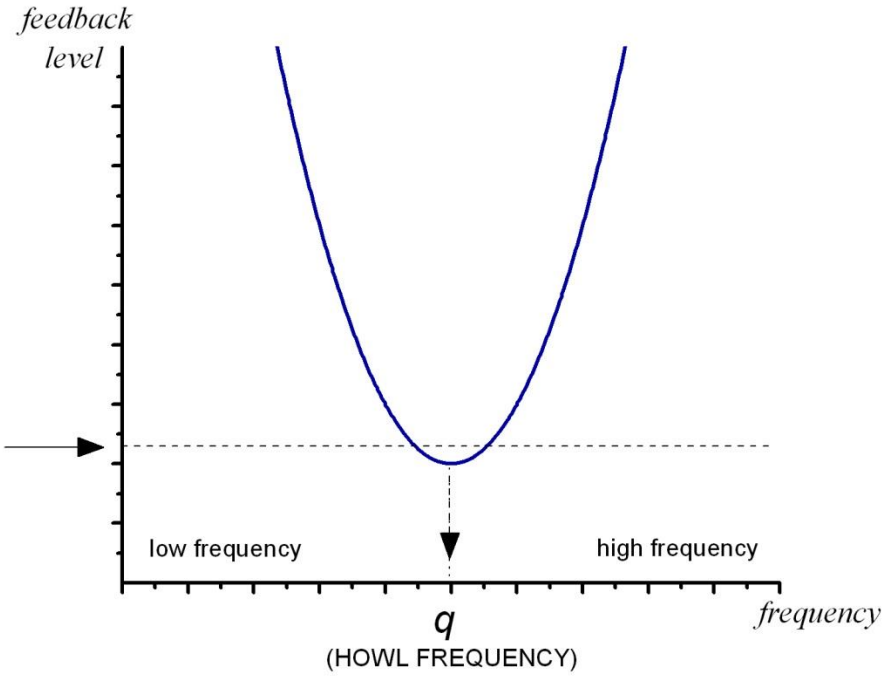




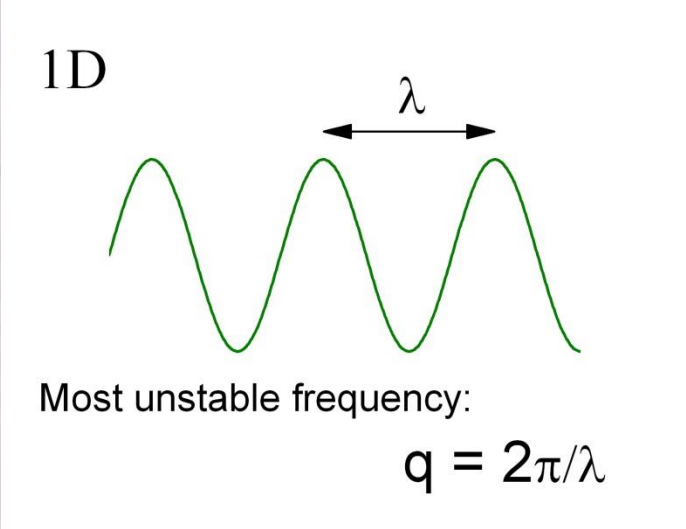


Barrio et al, Bull Math Biol 61, 483 1999

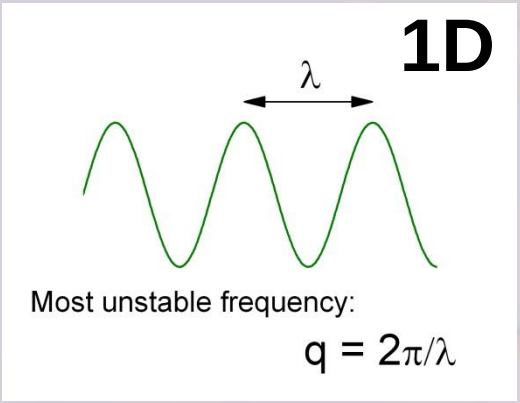
Audio feedback

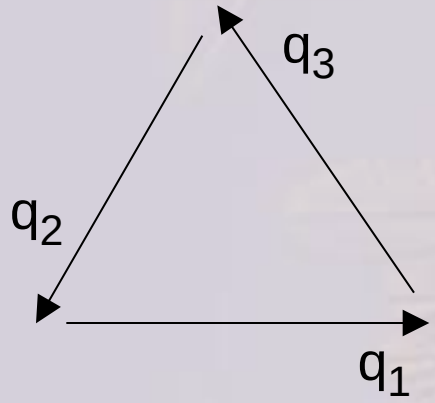


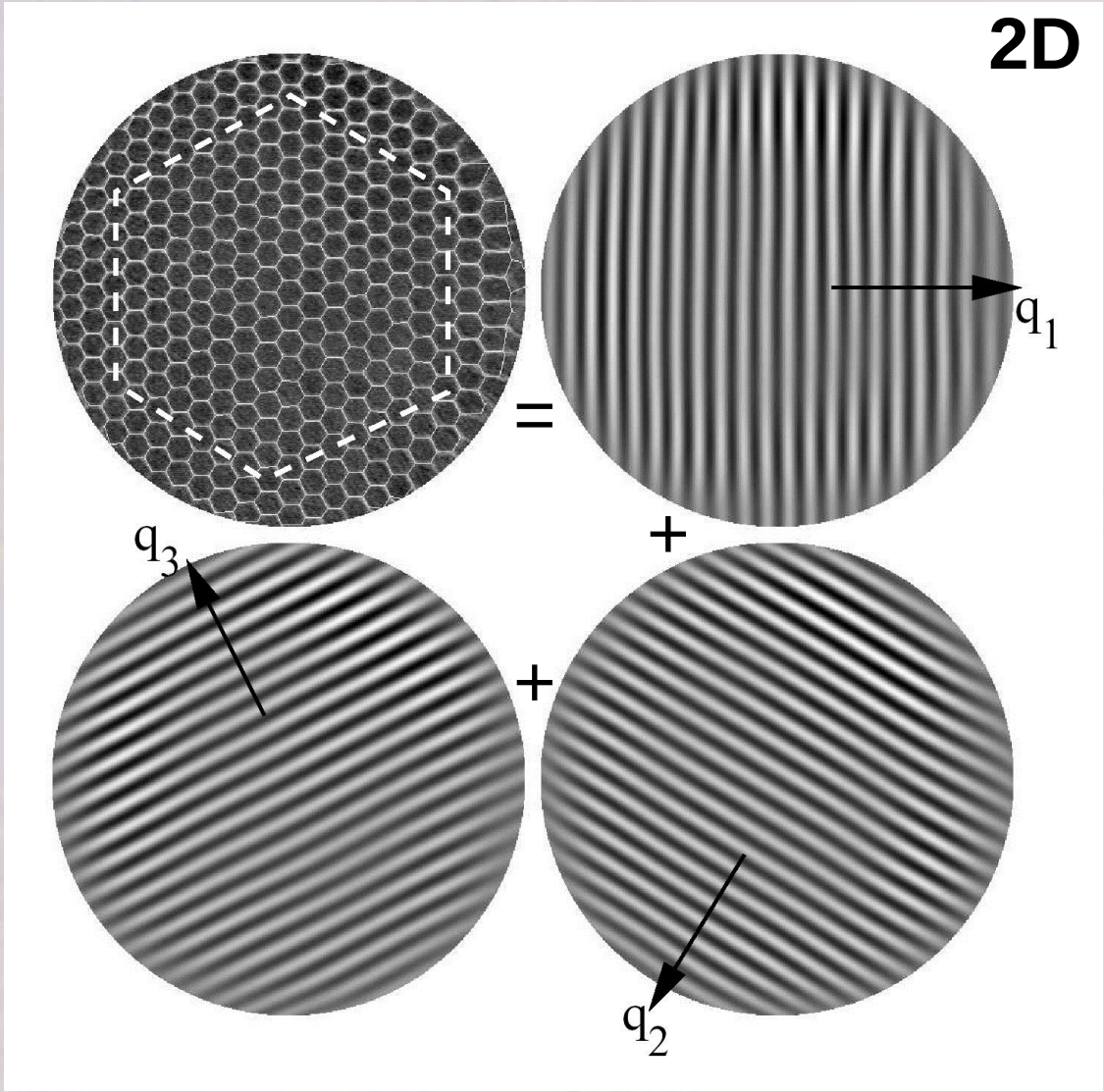
- System selects single simple pattern at *q*



Turing to stripes to hexagons







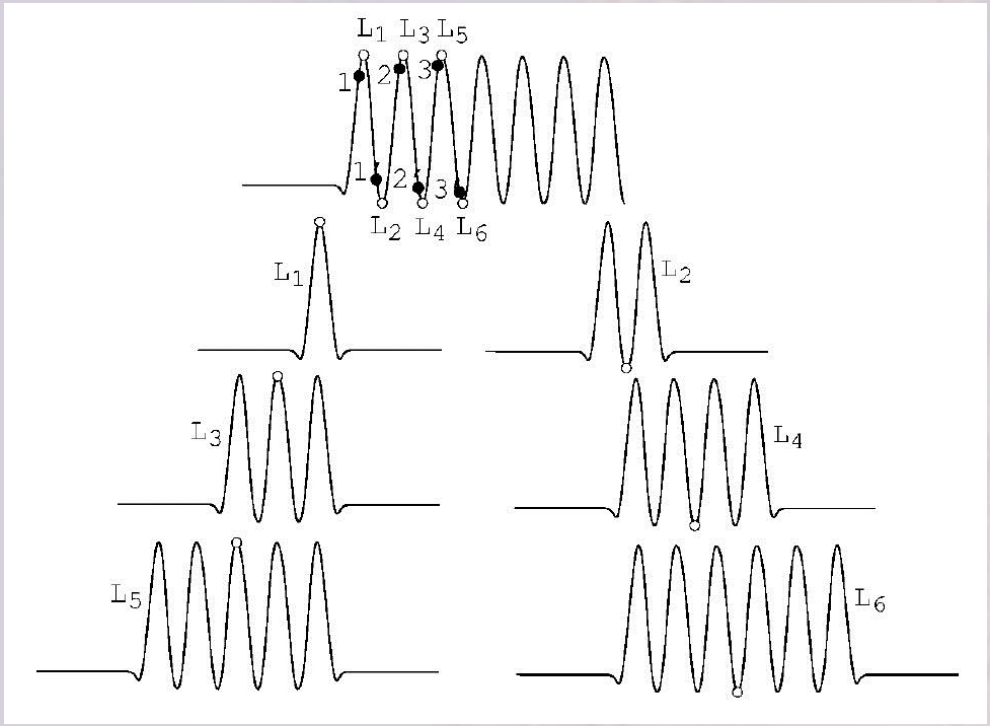
Hexagons to bright spots

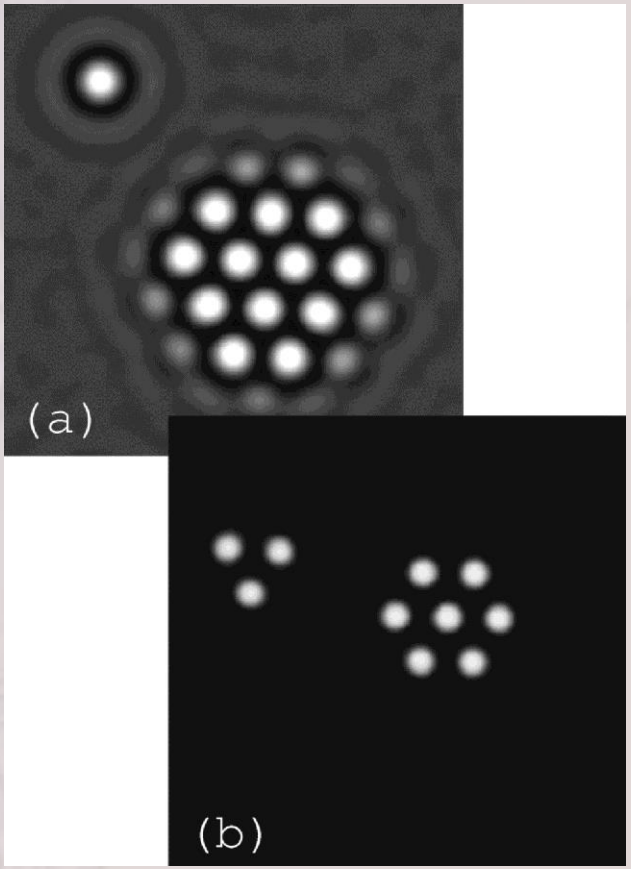
VOLUME 84, NUMBER 14 PHYSICAL REVIEW LETTERS 3 APRIL 2000

Stable Static Localized Structures in One Dimension

P. Coullet and C. Riera
INLN, 1361 Route des Lucioles, 06560, Valbonne, France

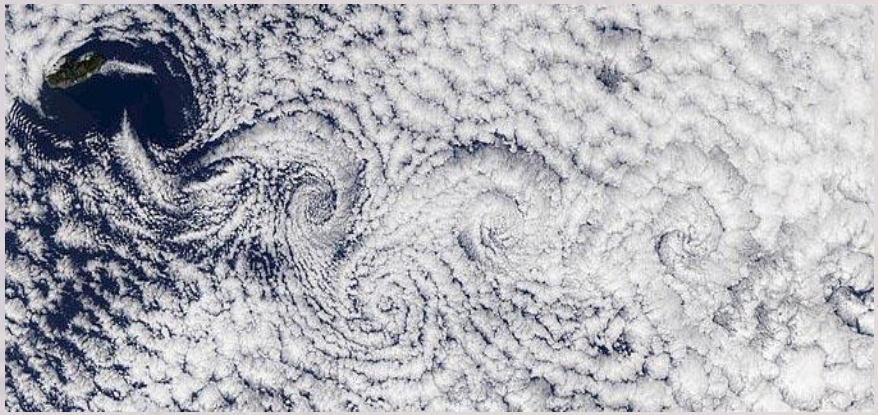
C. Tresser
IBM, P.O. Box 218, Yorktown Heights, New York 10598





next …

- Simple patterns vs. Complex patterns
- Feedback
- Perspective on fractals
- Origin of simple patterns
- Applications
  - photonics …
  - an origin of fractal patterns
  - global human population



from NASA'S VISIBLE EARTH

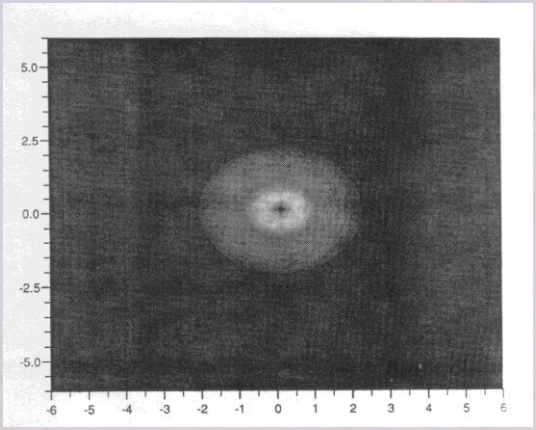
What use is complexity ?

→ models

What are models for ?

→
- prediction
- understanding

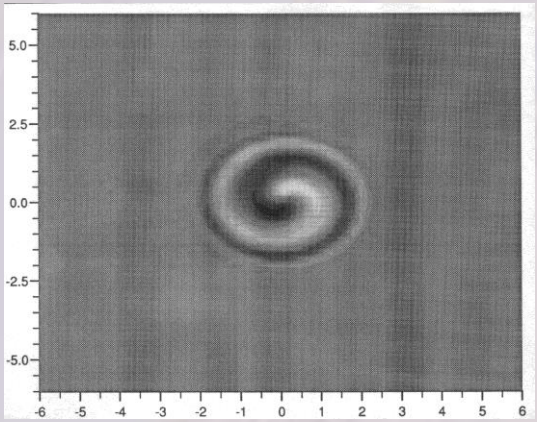
Prediction - entirely new phenonmena



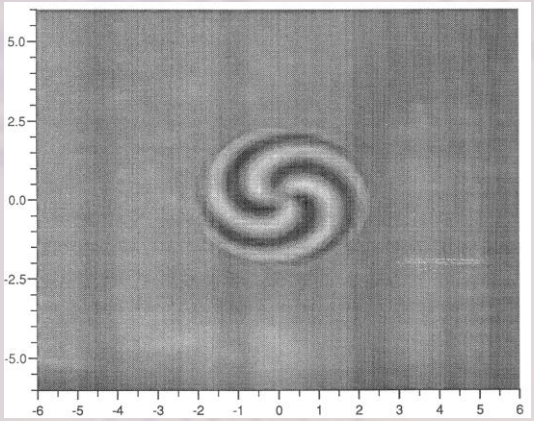
Beam X-section - intensity

Optical vortices

GS McDonald et al, Opt Commun 94, 469 (1992)

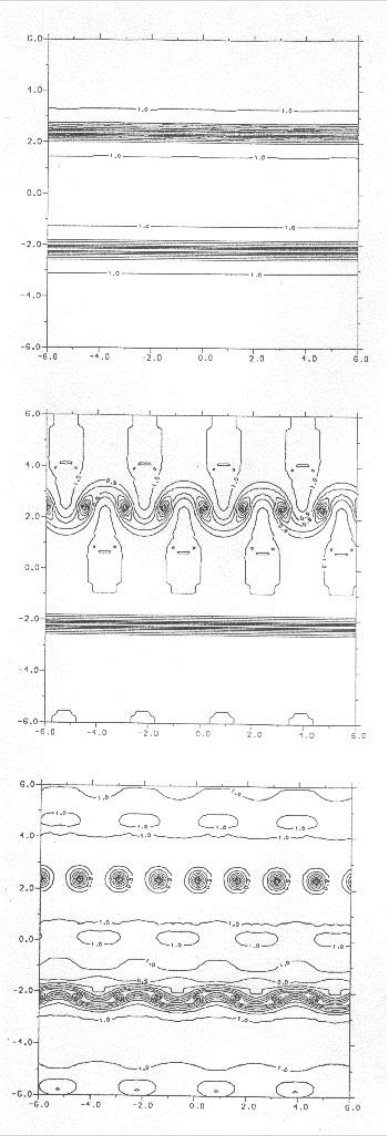


Beam X-section - phase



Beam X-section - phase

Prediction – birth of new structures

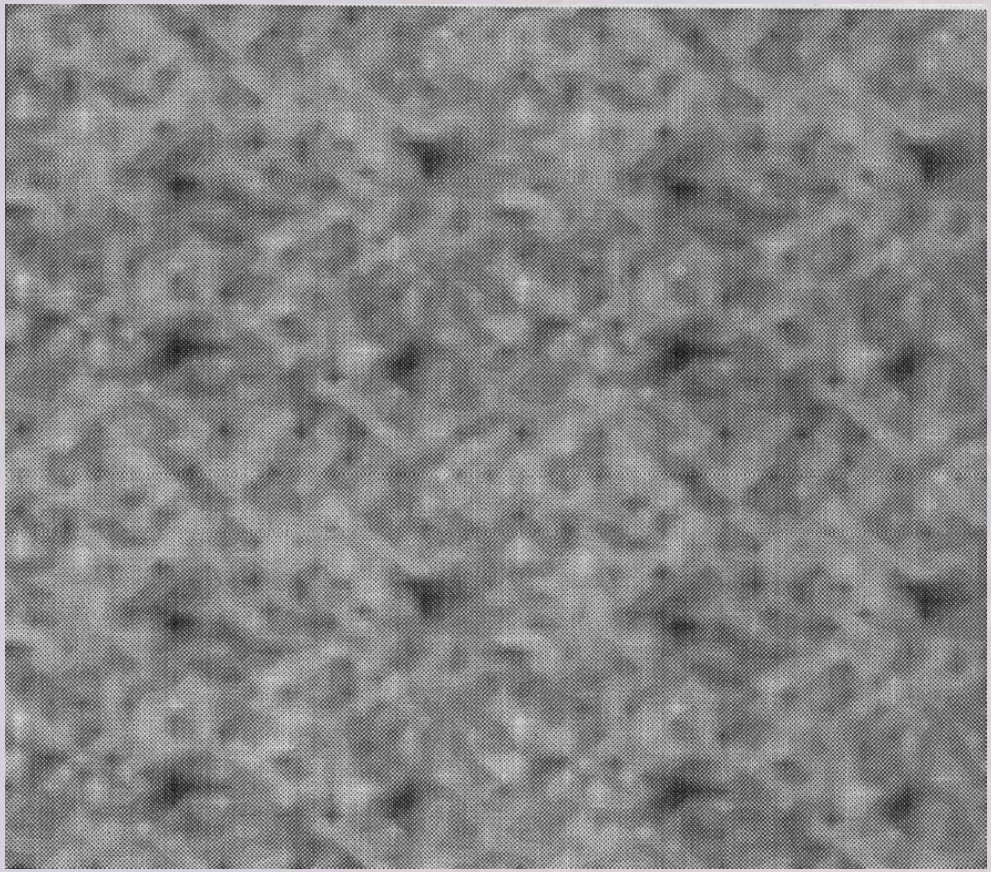


Dark lines (light intensity)

Spontaneous vortex streets

GS McDonald et al, Opt Commun 95, 281 (1993)

Prediction – configurations of vortices

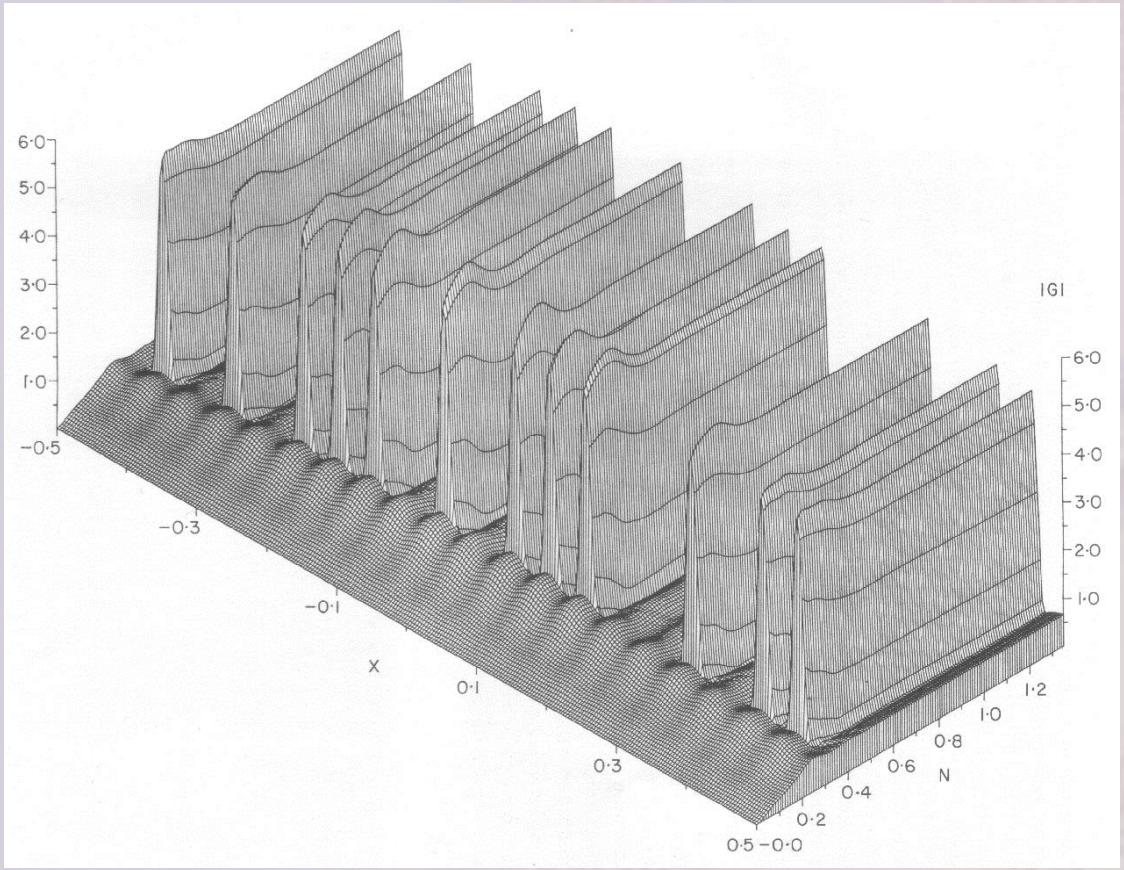


optical
vortex
array

(Patterns of
dark spots)

GS McDonald et al, Opt Commun 94, 469 (1992)

Prediction – novel applications

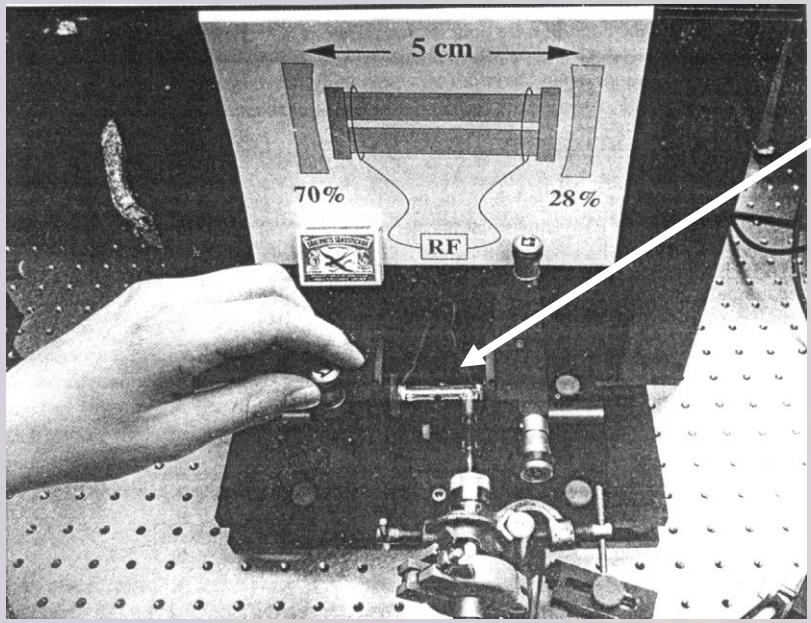


New computational architectures

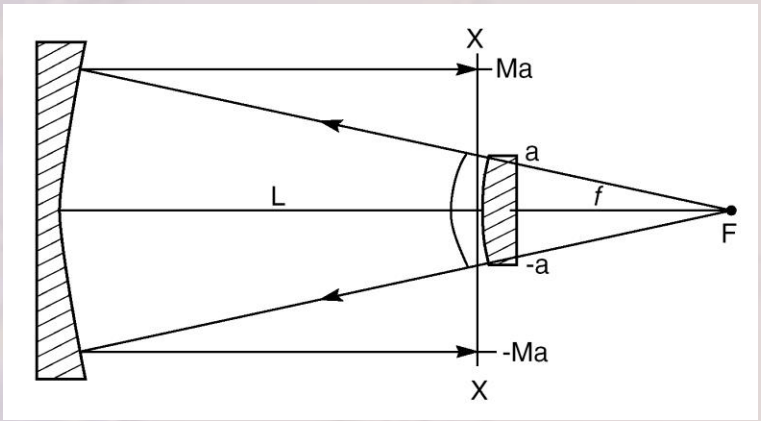
Pattern “0101011101111101011” in bright spots (solitons)

GS McDonald et al, JOSA B 7, 1328 (1990)

Fractal lasers - experiments

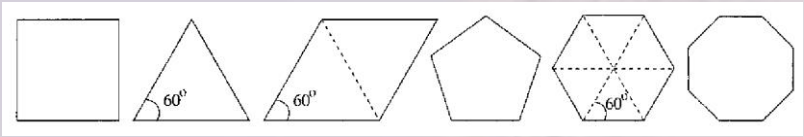


1. Miniature HeXe gas laser with 5 cm x 5 mm tube

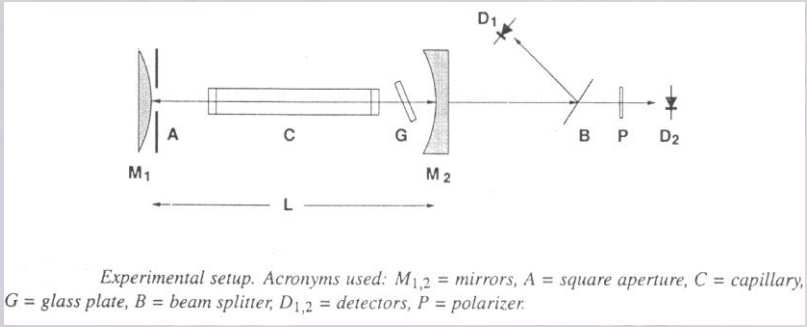


2. Mirrors (shaded) give unstable-cavity with beam magnification *M*

3. Light scatters on edges of smaller mirror having variety of effective shapes

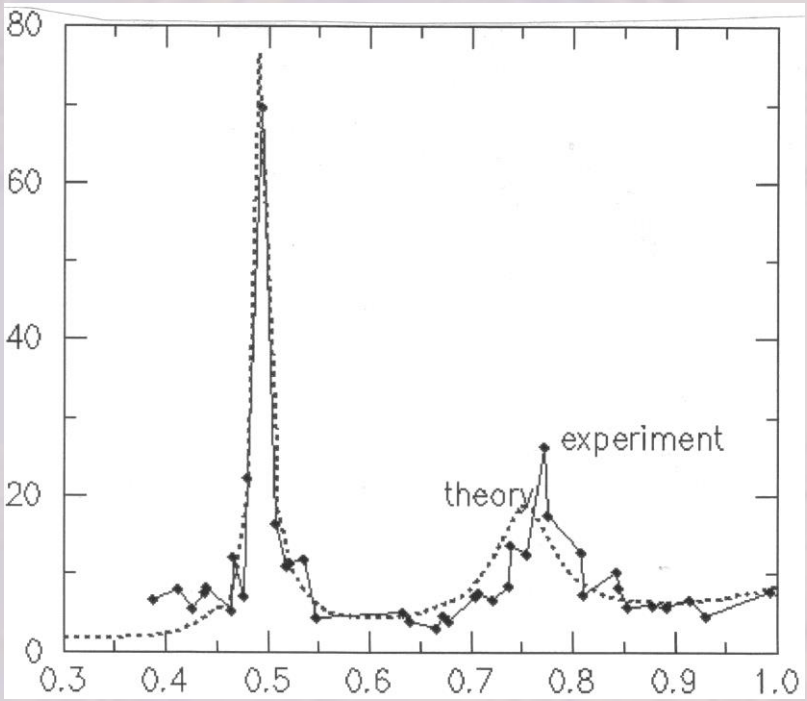


Typical experiment versus theory



Experimental setup. Acronyms used: $M_{1,2}$ = mirrors, A = square aperture, C = capillary, G = glass plate, B = beam splitter, $D_{1,2}$ = detectors, P = polarizer.

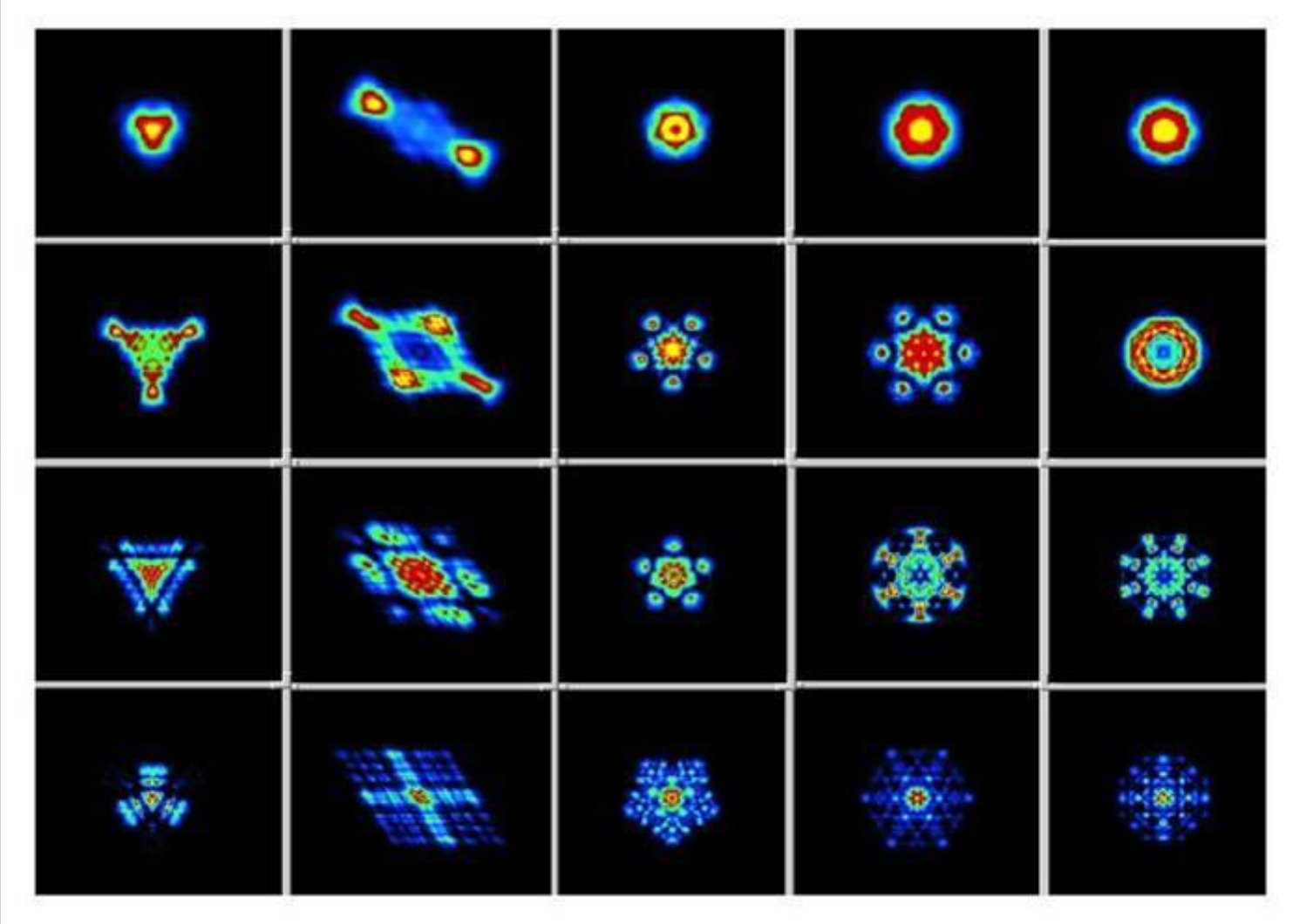
LASER
OUTPUT
CHARACTERISTICS



CAVITY PARAMETER VARIATION

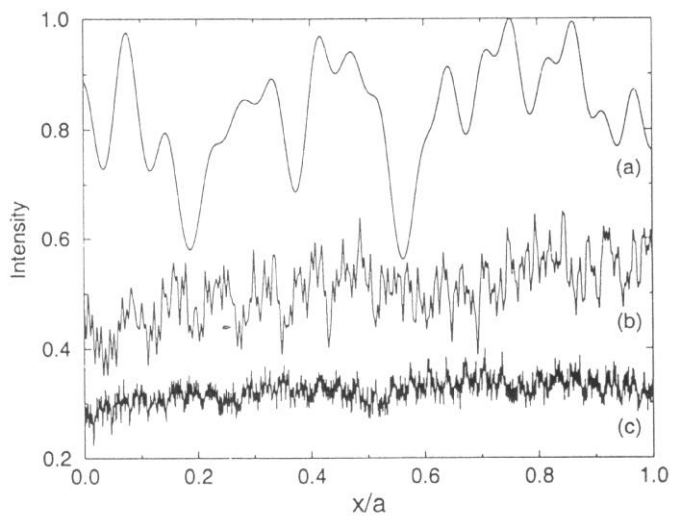
GS McDonald et al,
Opt Commun 164, 285 (1999);
Appl Optics 38, 6874 (1999)

Cross-sections of laser beam profiles



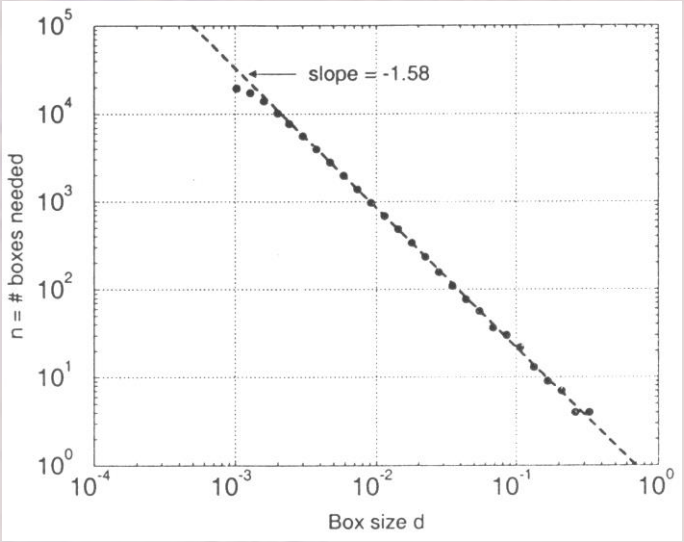
GS McDonald et al, NATURE 402, 138 (Nov, 1999); JOSAB 17, 524 (2000)

Confirmation of fractal character

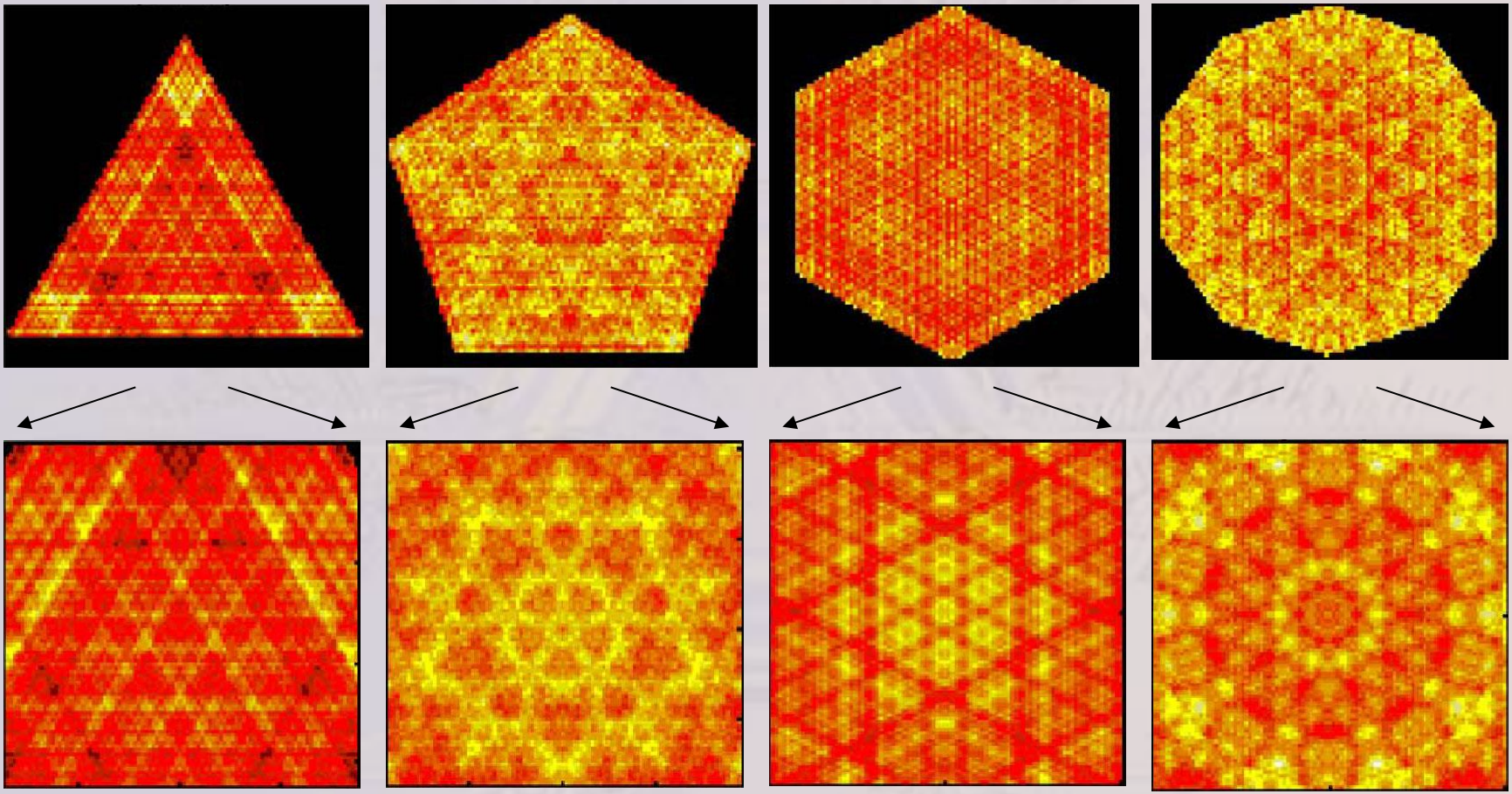


Output light profiles
for 3 different cavities

Straight-line log-log
plot (ruler length)
→ fractal



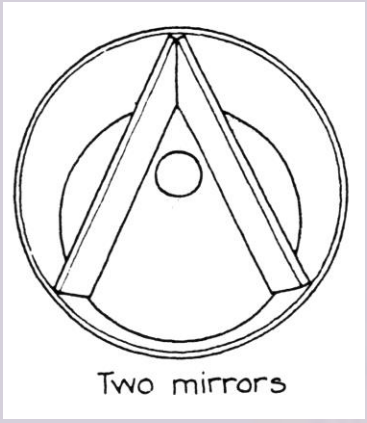
Recent developments

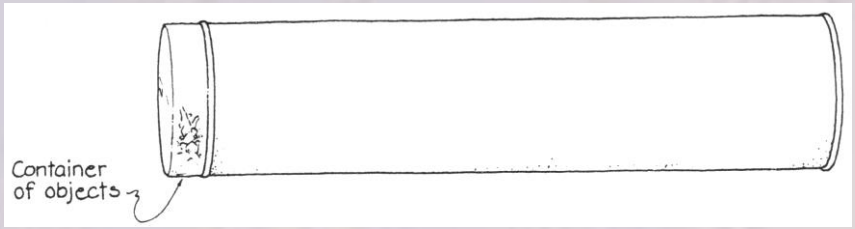


New Fresnel diffraction formulations allow calculation of arbitrary level of detail

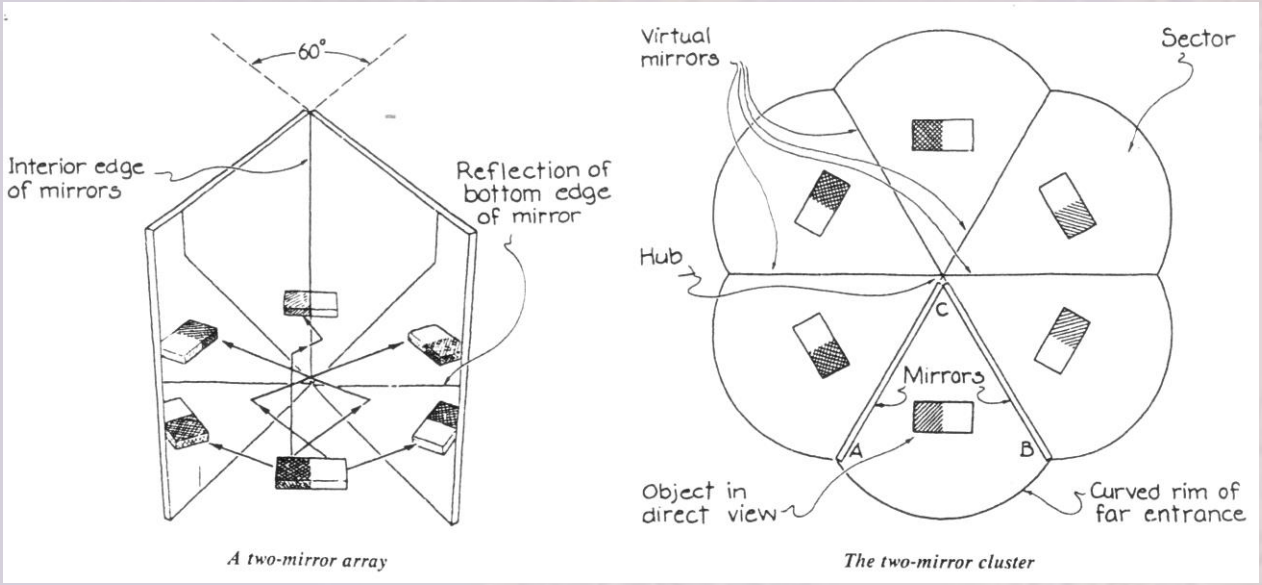
Huang, Christian, McDonald, JOSA A 23, 2768 (Nov, 2006)

Kaleidoscope laser …



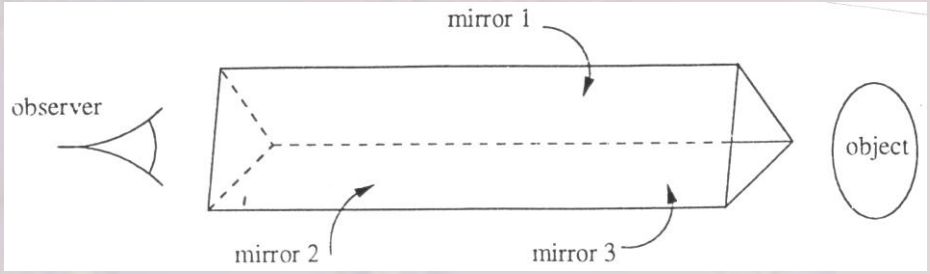


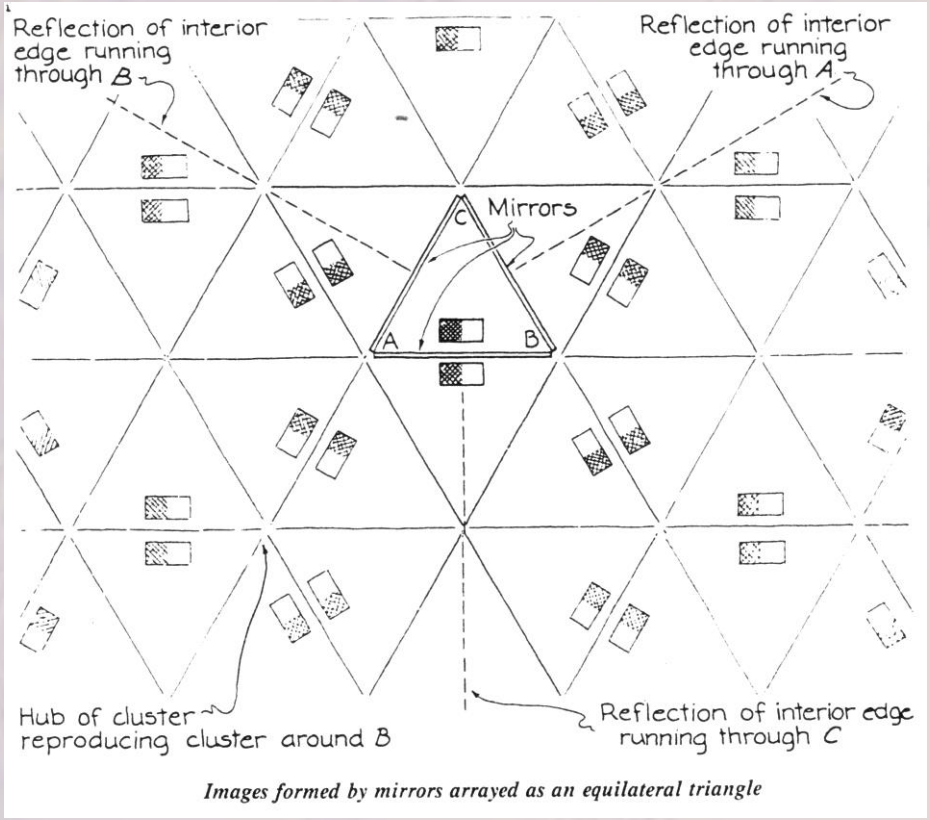
David Brewster
(born 1781)



J Walker, *The Amateur scientist*, Sci Am 253, 124 (1985)

3-mirror kaleidoscope





J Walker,
The Amateur scientist,
Sci Am 253, 124 (1985)

next …

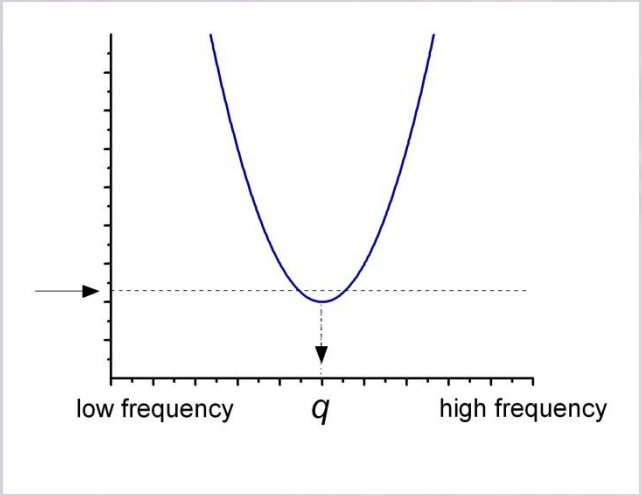
- Simple patterns vs. Complex patterns
- Feedback
- Perspective on fractals
- Origin of simple patterns

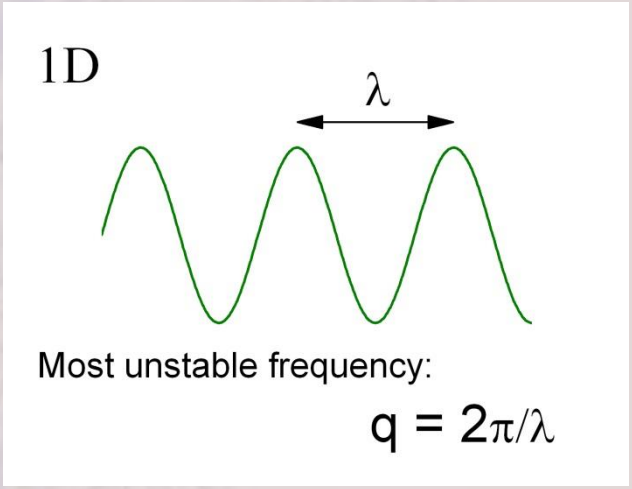
- Applications
  - photonics
  - an origin of fractal patterns …
  - global human population

From Turing instability to fractals ?

Two signatures of complexity:

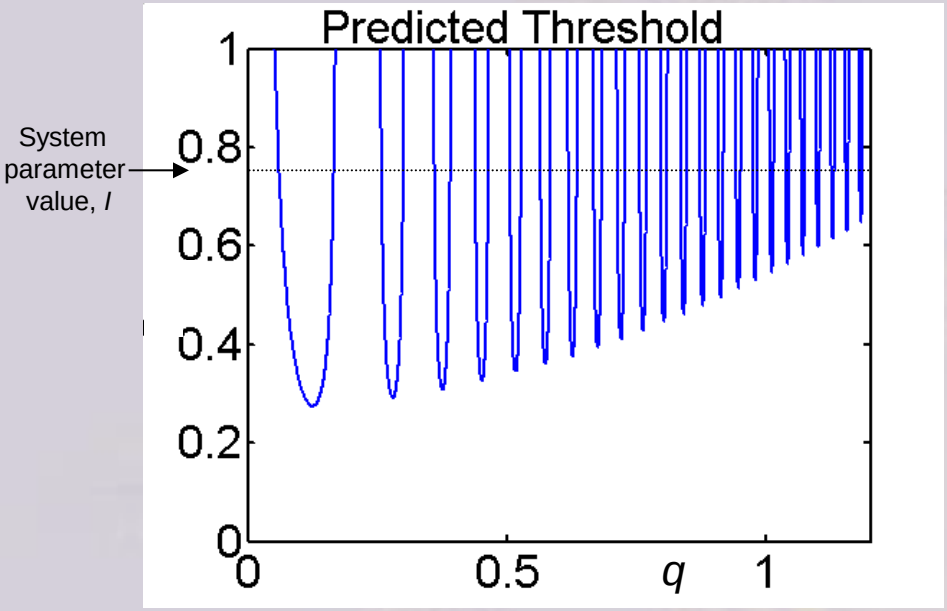
- Simple pattern formation
  (emergence of *one* characteristic scale)
- Fractal pattern formation
  (emergence of *scale-less* structures)





TURING INSTABILITY
(simple pattern formation)

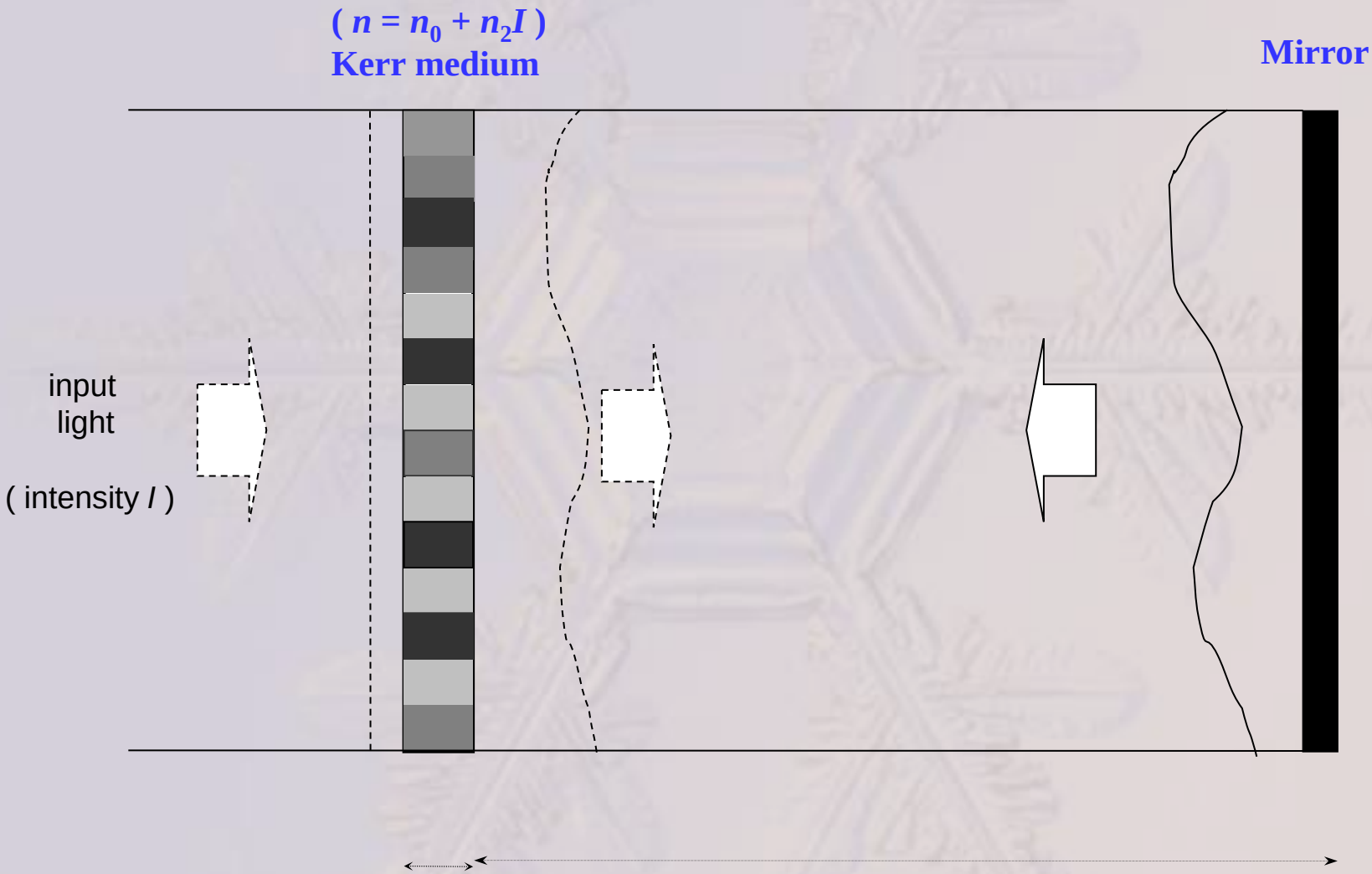
Multi-Turing instability



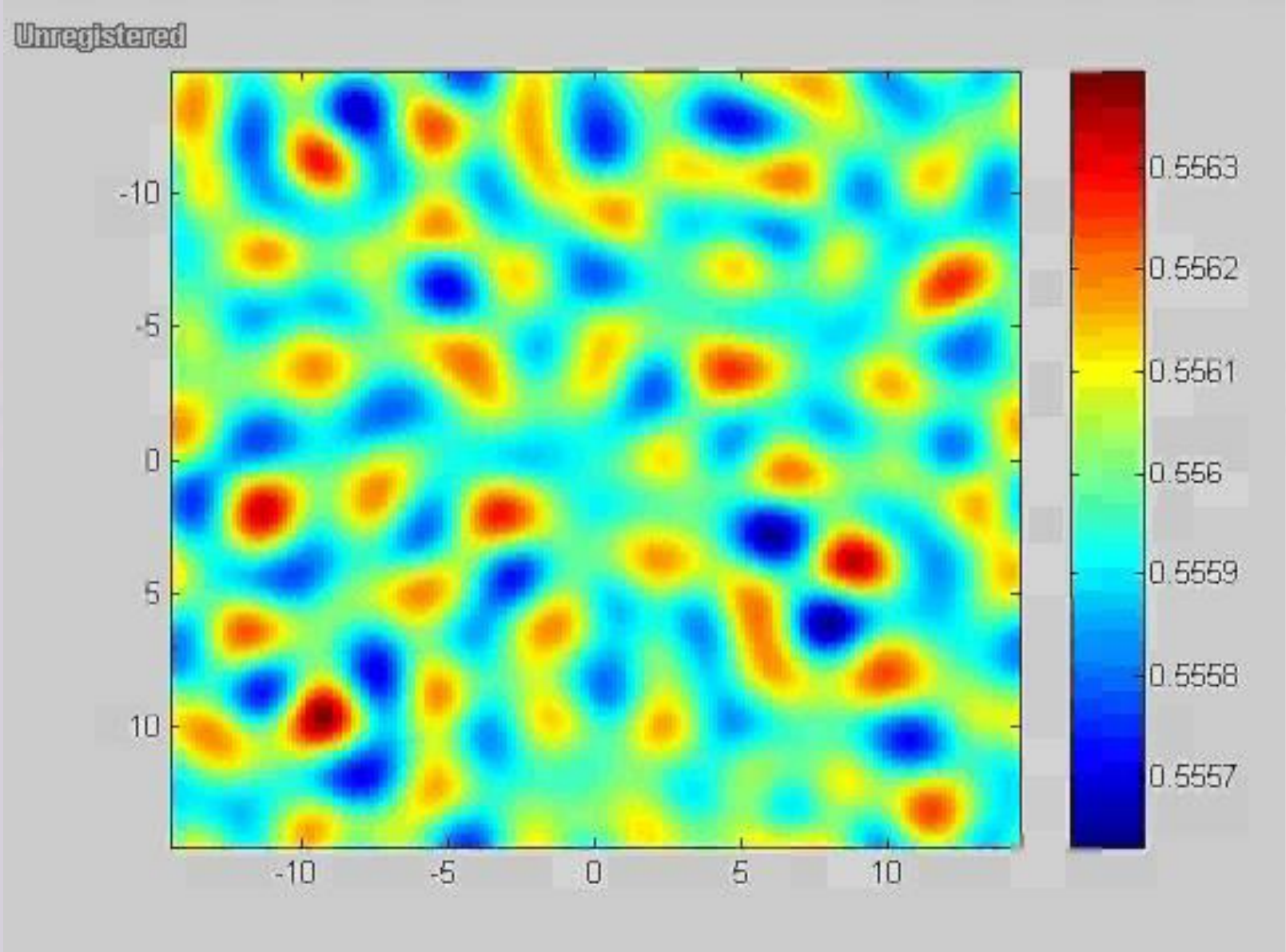
? fractals ?

- Wide class of systems actually possess *many* Turing minima
- Analysis generally assumes system parameter set close to first minimum

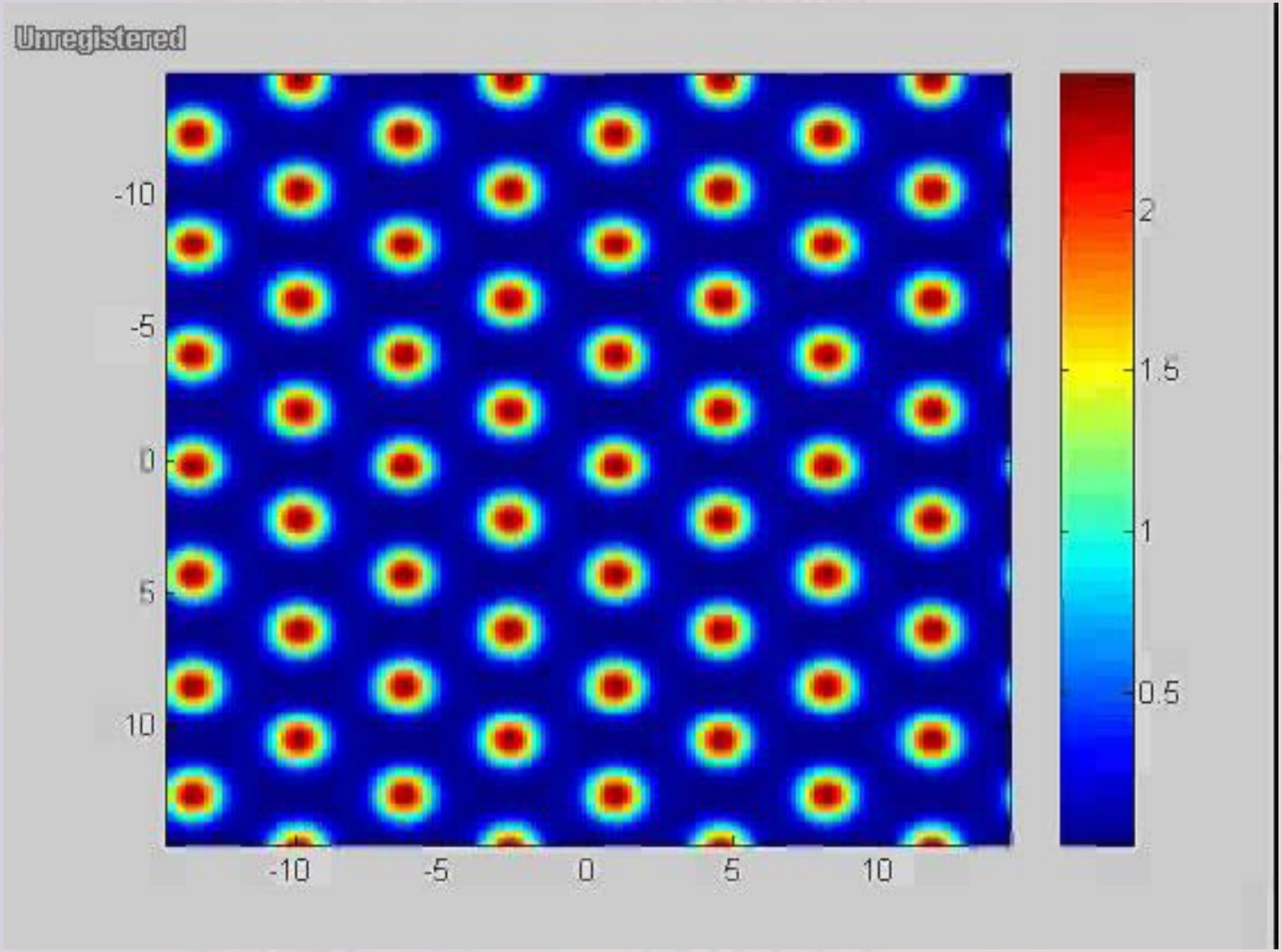
Simple optical system



With filtering (1st min only)



Without filtering (all min's)



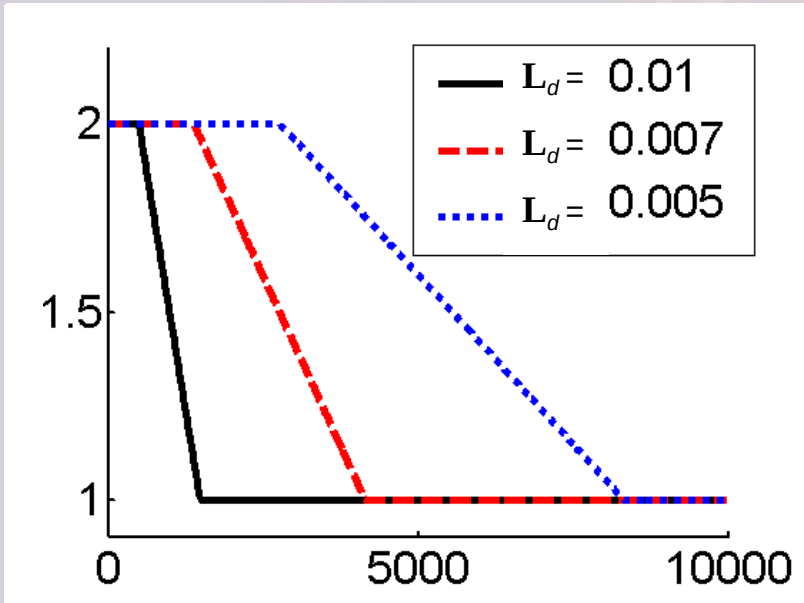
Huang & McDonald, Phys Rev Lett 94, 174101 (2005)

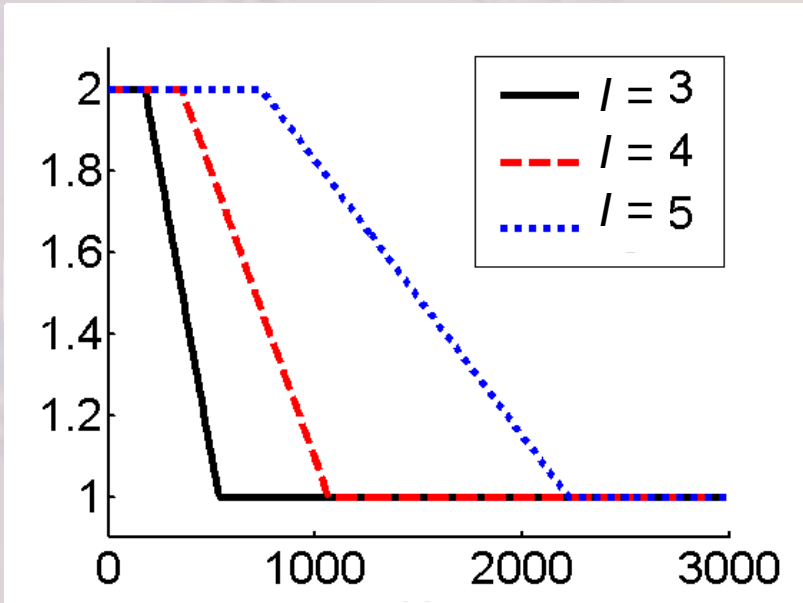
Transition at small scales

medium diffusion, $\mathbf{L}_d$

input light intensity, I

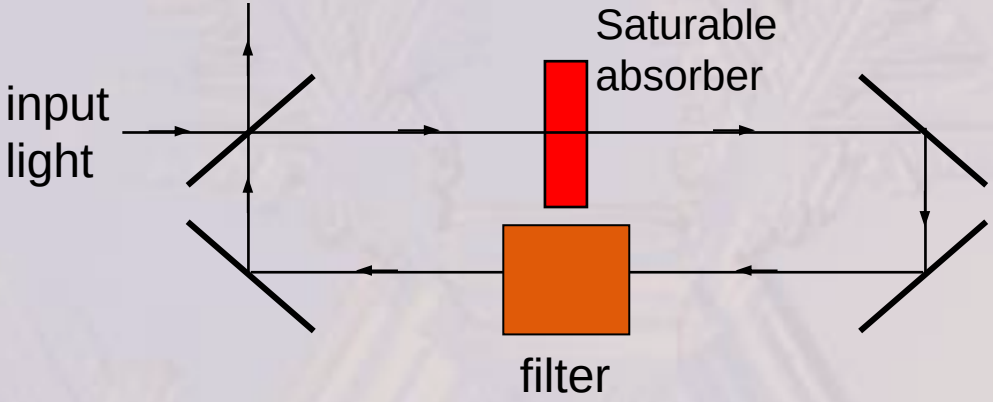
FRACTAL DIMENSION, D

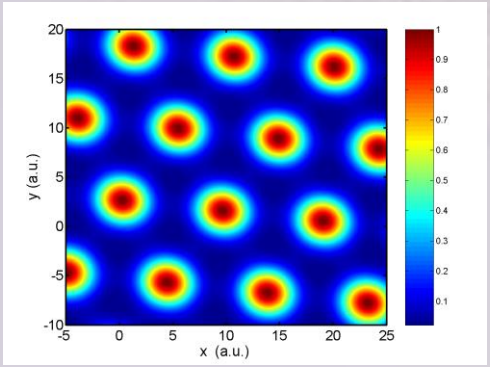


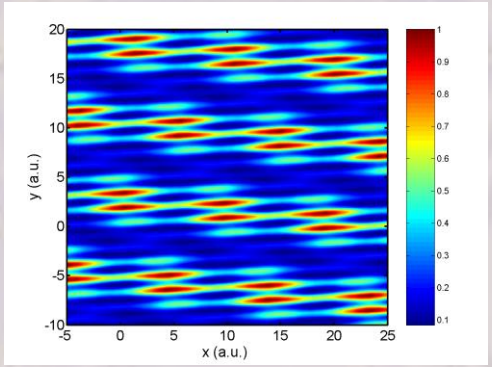


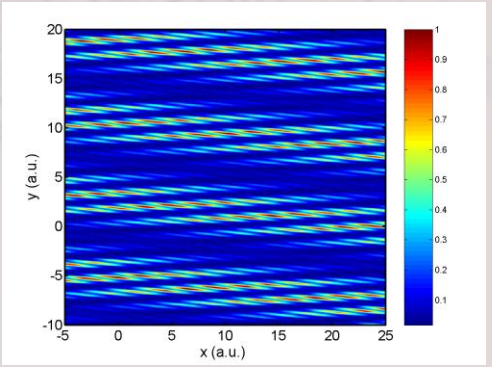
SPATIAL FREQUENCY, q

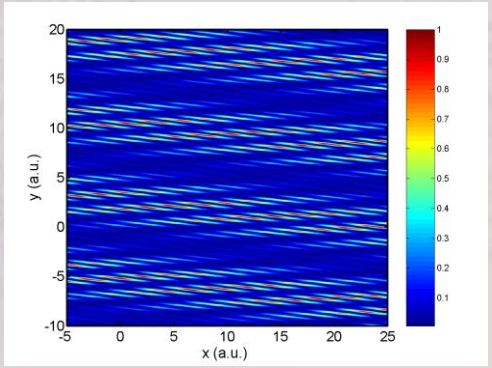
Example of other systems











Huang, Christian & McDonald, submitted to Opt Lett

next …

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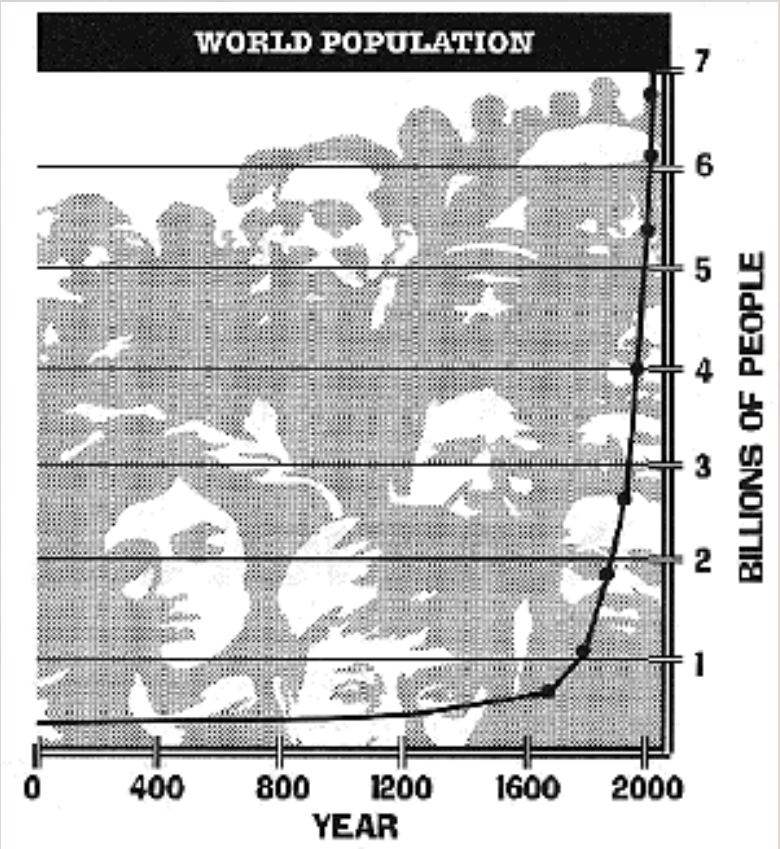
  - photonics
  - an origin of fractal patterns
  - global human population …



MESEROVIC, MIHAJLO & PESTEL (1974)

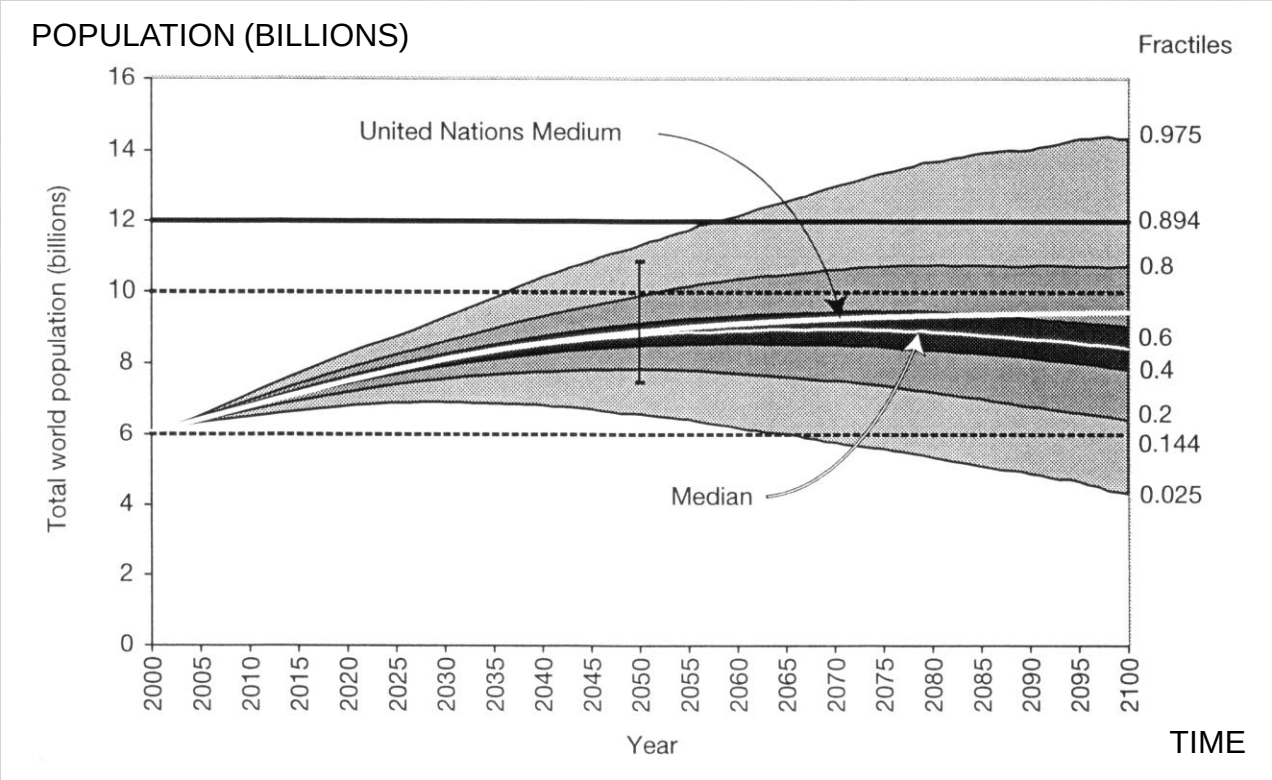
Global human population

letters to nature

The end of world population growth

Wolfgang Lutz*, Warren Sanderson*† & Sergei Scherbov‡

v412, 543 (2001)



but this is a complex system

Conventional modelling tends to be error-prone

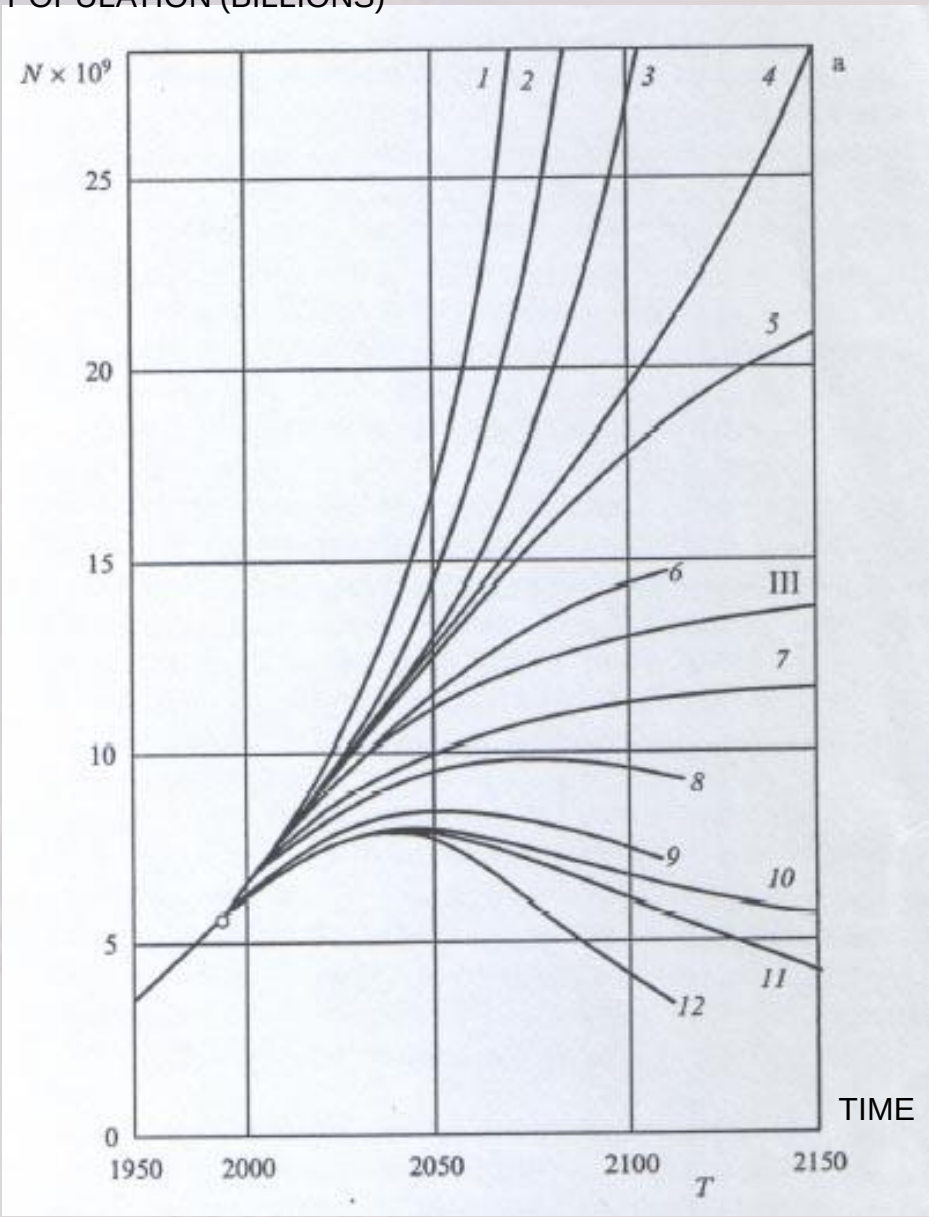


Figure 7. (a) World population projections by UN [27] and IIASA [28]. *1* — constant fertility, *2* — constant rate of growth, *3* — III World crisis, *4* — high UN, *5* — medium high UN, *6* — low decrease of fertility, *7* — medium decrease of fertility, *8* — slow decrease of mortality, *9* — constant mortality, *10* — medium low UN, *11* — low UN, *12* — rapid decrease of fertility, III — Model III, ∘ – present. (b) Changing age distribution during the demographic transition. *13* — less than 15 years, *14* — older that 65 years, according to medium UN projection [27].

SP Kapitza, Uspekhi 29, 57 (1996)

People as particles

Underlying patterns

POPULATION

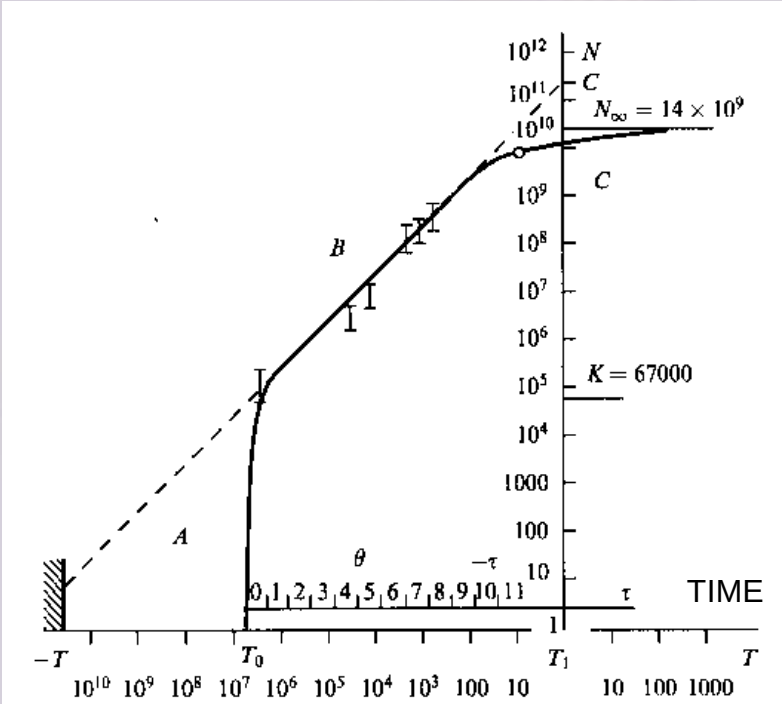


Figure 5. World population since the origin of humankind to the foreseeable future. Data [5, 26] and Coppens [23], $\theta = \ln t'$ (22), – – – – (2), ○ — present.

LOCAL POPULATION GROWTH

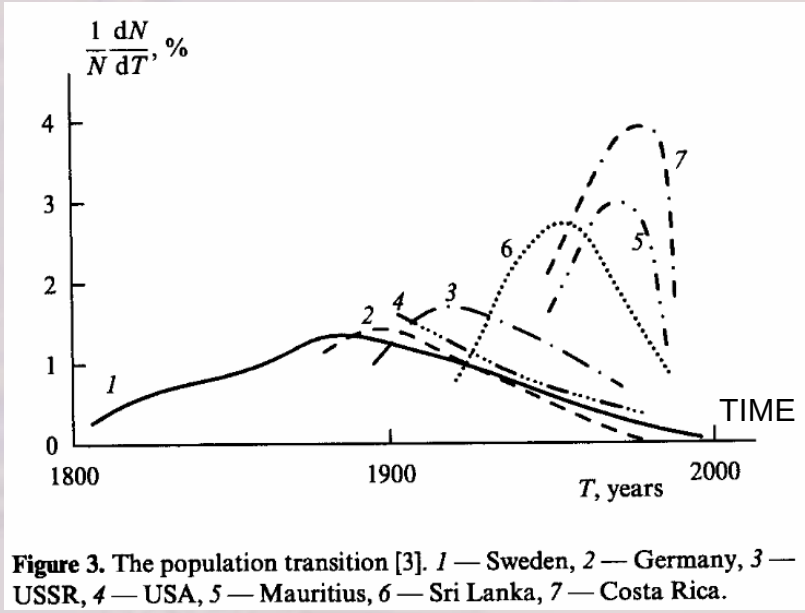


Figure 3. The population transition [3]. *1* — Sweden, *2* — Germany, *3* — USSR, *4* — USA, *5* — Mauritius, *6* — Sri Lanka, *7* — Costa Rica.

SP Kapitza, Uspekhi 29, 57 (1996)

Complexity global modelling

- Simple models give good agreement
- Lend understanding
- Can have only 2 key parameters …
  - Large scale: coherent population unit
  - Small scale: average lifetime of everyone who has ever lived … = 42 (!)

Conclusion

Stephen Hawking prediction:

this 21st century will be

“the century of complexity”