Two-colour optical fields: modulational instabilities and vector spatial solitons

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Abstract

Two-colour nonlinear light beams comprise a pair of optical waves at two well-separated temporal frequencies, and which are coupled together by way of the refractive properties of the host medium [1–3]. Exotic electromagnetic structures of this kind involve a complex and subtle interplay between many distinct feedback loops, each of which is a combination of linear (two-dimensional diffraction) and nonlinear (self- and cross-focusing) processes. The interaction between the constituent waves is strongest when they overlap in the propagation plane. Under the right conditions, the feedback loops may reach an equilibrium point whereupon the system can sustain stationary localized states – *vector spatial solitons*. Such waves play a key role in our understanding of a wide range of nonlinear optical phenomena. In addition to their fundamental theoretical interest, two-colour vector spatial solitons have huge potential for exploitation in future photonic device architectures that operate on multi-frequency principles [4].

Classic analyses of vector spatial solitons have been limited by assumptions of beam paraxiality [1–4]. While this conventional approach proffers a model with a simple nonlinear-Schrödinger form, its adoption imposes strong restrictions on the physical regimes that may be accurately described. Our research opens up fresh avenues of exploration into two-colour photonics by going beyond well-known paraxial wave optics. Recently, we have been able to take the first steps towards a deeper understanding of two-colour light fields by relaxing the ubiquitous slowly-varying envelope approximation, and dealing instead with a more general pair of coupled nonlinear Helmholtz equations [5]:

$$\kappa\frac{\partial^2 u_1}{\partial\zeta^2}+i\frac{\partial u_1}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u_1}{\partial\xi^2}+p\left(|u_1|^2+\sigma|u_2|^2\right)u_1=0, \qquad \kappa\frac{\partial^2 u_2}{\partial\zeta^2}+i\beta\frac{\partial u_2}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u_2}{\partial\xi^2}+p\alpha\left(|u_2|^2+\sigma|u_1|^2\right)u_2=0. \tag{1a,b}$$

Briefly, u_1 and u_2 are the dimensionless electric field envelopes at the two frequencies, while (ξ, ζ) are the transverse and longitudinal coordinates (normalized with respect to the waist and diffraction length of a reference Gaussian beam, respectively). The small parameter $\kappa << O(1)$ quantifies beam waist compared to the optical wavelength, and $p = \pm1$ flags a focusing/defocusing nonlinearity. Coupling between the two fields is parameterized by σ, while α and β are related to the dispersive properties of the host medium (ratios of wavelengths and linear refractive indexes). The paraxial model [1,2] is obtained by neglecting the first term in each equation.

We will present, to the best of our knowledge, the first analysis of two-colour Helmholtz light fields. Plane wave modulational instability (MI) calculations [6] have been made through linearization techniques [see Fig. 1(a)]. While Agrawal's paraxial analysis [7] holds only for the special case $\sigma = 2$, our generalized approach involves two-fold novelty: (i) consideration of arbitrary values of the coupling parameter σ, and (ii) inclusion of Helmholtz-type nonparaxiality. Extensive simulations of spontaneous MI have tested, and confirmed, our theoretical predictions of the resilience of two-colour plane waves to small perturbations [see Fig. 1(b)].

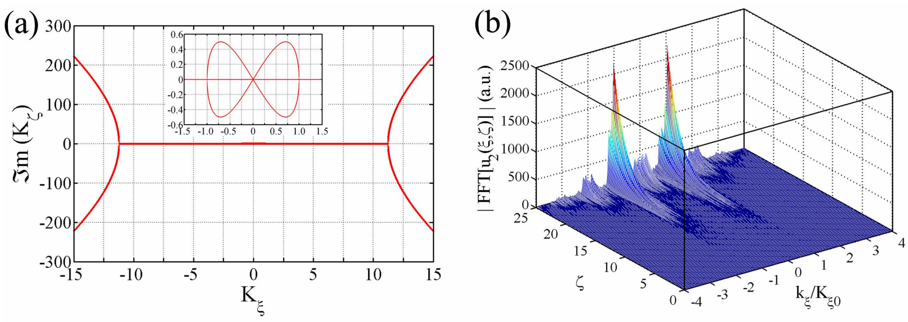

Figure 1. (a) Typical induced-MI spectrum for a two-colour plane wave with $\sigma > 1$. The most unstable spatial frequency, referred to as $K_{\xi0}$, corresponds to the peak of the "bow-tie" (inset) of the MI gain curve. (b) Evolution of the spatial spectrum of one component (here, u_2) in an unstable two-colour plane wave perturbed by a 1% level of background noise. The spatial frequencies that grow preferentially during the early stages of propagation corresponds to $\pm K_{\xi0}$, which confirms the predictions of our analysis.

In-depth analysis of model (1) has also uncovered four families of exact analytical vector soliton: *bright-bright* and *bright-dark* for a focusing nonlinearity, *dark-bright* and *dark-dark* for defocusing. It is worth emphasising that bright-dark and dark-bright solutions are physically distinct from each other (for instance, they possess notably different stability properties). Each vector soliton family has co- and counter-propagation classes that are related by geometrical transformation; this type of bi-directionality is a natural consequence of our model, where the $\kappa\partial^2/\partial\zeta^2$ operators have been retained. Simulations and analysis have also been used to predict the robustness of the new spatial solitons (see Fig. 2).

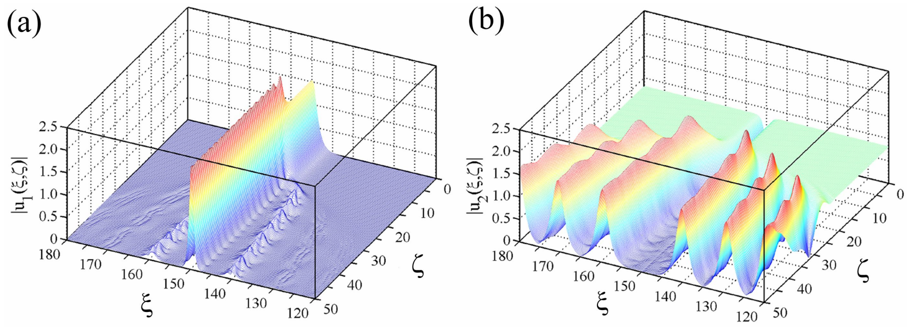

Figure 2. Modulational instability of the bright-dark Helmholtz soliton family [bright component in (a), dark component in (b)] in a focusing Kerr medium. Instability develops initially on the plane-wave background of the dark component, leading to filamentation. Nonlinearity provides a mechanism whereby this instability subsequently feeds through the system to destabilize the bright component.

In conclusion, we have proposed a more sophisticated model for describing two-colour light fields in planar waveguides using a nonlinear Helmholtz-type formalism. We have unravelled the MI properties of this generalized system, and the predictions of our fully-2nd-order calculation have been borne out by extensive simulations. We note, in passing, that the algorithm routinely deployed to integrate single-colour Helmholtz equations (see Ref. 8) had to be modified non-trivially in order to solve Eqs. (1a) and (1b) numerically. New families and classes of exact analytical Helmholtz vector soliton have been derived. These solutions are endowed with a wide range of physically desirable properties that are absent from their paraxial counterparts [1–3], and their stability properties have been mapped by combining linear analysis with computer simulations.

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