Complexity and Fractality in Kaleidoscope Laser Eigenmodes

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Abstract: Kaleidoscope lasers are investigated using virtual source modelling. We calculate families of eigenmodes for cavities with arbitrary parameters, and consider the implications of non-trivial transverse symmetry for fractal dimension.
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1. Kaleidoscope lasers

Unstable cavity lasers exhibit a plethora of bizarre and potentially exploitable phenomena that have fascinated researchers over the last four decades. In particular, the innate capacity of such simple systems for generating complex light patterns continues to attract wide and sustained interest. In the late 1990s, Karman and Woerdman [1] found that the eigenmodes of one-dimensional (1D) confocal resonators are *fractals* – patterns that exhibit proportional levels of detail spanning decimal orders of scale. The fractality of these self-reproducing mode profiles was later shown to originate in the interplay between cavity magnification and diffraction at the outer boundary of the system (i.e., the feedback mirror) [2].

A kaleidoscope laser [3] is an intuitive generalization of a classic unstable strip resonator [1] to fully-2D transverse geometries, where the defining aperture has the shape of a regular polygon. The non-orthogonal edges of this element have a profound impact on the structure of the cavity eigenmodes, which exhibit striking complexity and beauty. Most obviously, N-sided regular-polygon boundary conditions impose N-fold rotational symmetry on the intensity pattern. Transverse symmetry also has a strong influence on excess-noise characteristics [4].

In this presentation, we will report on the first detailed analysis of kaleidoscope lasers through accommodation of arbitrary equivalent Fresnel number N_{eq} (which quantifies the cavity aspect ratio) and round-trip magnification M. All previous analyses have been valid strictly only in restrictive parameter regimes: typically $N_{eq} = O(1)$ (where fully-numerical ABDC paraxial matrix modelling, in combination with Fast Fourier Transforms, can be used [5]) or $N_{eq} >> O(1)$ (in which case asymptotic approximations may be applied [6]). To facilitate a complete description of intermediate parameter regimes, we have deployed a semi-analytical approach that combines a fully-2D generalization of Southwell's Virtual Source (2D-VS) method [7] with exact mathematical descriptions of edge-wave (Fresnel) patterns from polygonal apertures [8] [see Fig. 1(a)].

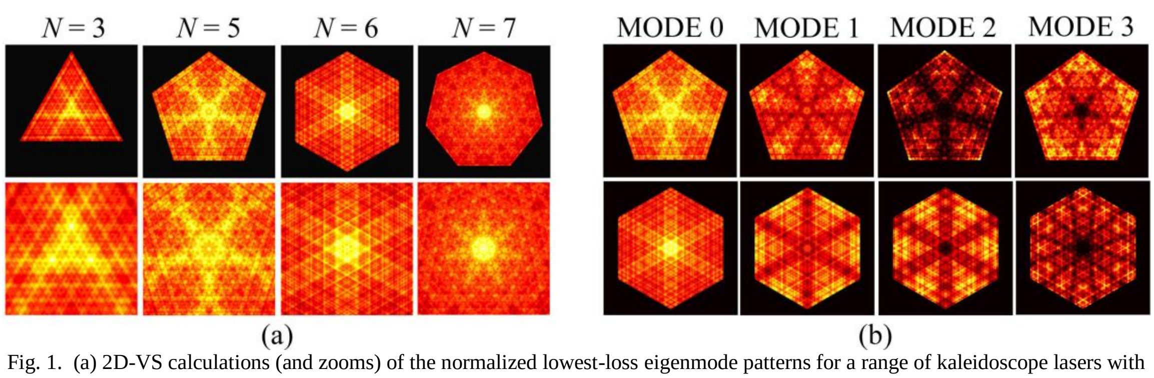

Fig. 1. (a) 2D-VS calculations (and zooms) of the normalized lowest-loss eigenmode patterns for a range of kaleidoscope lasers with parameters $N_{eq} = 30$ and $M = 1.5$. (b) First four modes for pentagonal (upper panes) and hexagonal (lower panes) feedback mirrors.

One key advantage of our 2D-VS method is that a single calculation allows one to construct both the lowest-loss mode *and* entire families of higher order modes (ABCD matrix modelling generates only a single pattern per application) [see Fig. 1(b)]. Another is that any desired section of the total mode pattern may be calculated, allowing small-scale details of the pattern to be revealed without loss of resolution (the ABCD matrix method computes only the global pattern and does not permit 'zooming').

2. Two-dimensional virtual source theory

The virtual source method [6] unfolds an unstable cavity into its equivalent sequence of N_S = log(250N_{eq})/log$M$ virtual apertures. Any eigenmode can then be constructed from a combination of edge-waves from each aperture, plus a plane-wave contribution. In scaled units, the mode pattern $V(\mathbf{X})$ is given by the linear superposition

$$V(\mathbf{X}) \propto \frac{E_{N_S+1}(\mathbf{X_C})}{\alpha^{N_S}(\alpha-1)} + \sum_{m=1}^{N_S} \alpha^{-m} E_m(\mathbf{X}), \tag{1}$$

where $\mathbf{X}$ denotes an appropriate set of transverse coordinates, $\mathbf{X_C}$ is any point on the boundary of the feedback mirror, $E_m(\mathbf{X})$ is the edge-wave pattern arising from the m^{th} virtual aperture [7], and the weighting factor α is interpreted as the complex mode eigenvalue (obtained by solving an associated polynomial equation).

3. Circular feedback mirror

The convergence properties of kaleidoscope laser characteristics (e.g., eigenvalue spectra and the mode patterns themselves) in the circular limit ($N \to \infty$) will be discussed (see Fig. 2).

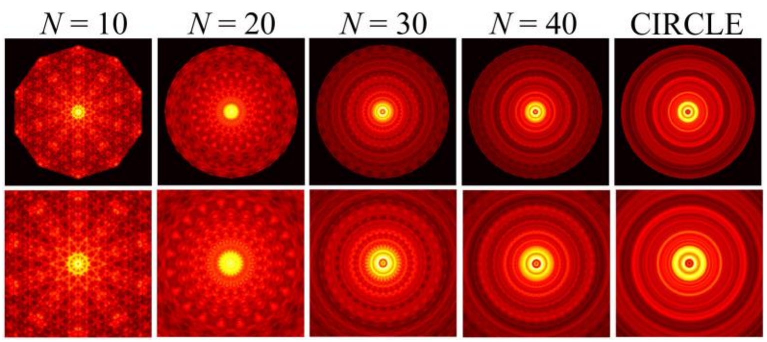

Fig. 2. Convergence of the lowest-loss kaleidoscope laser (cavity parameters N_{eq} = 30 and M = 1.5) toward its circular counterpart.

The transition towards circularity is fraught with physical subtleties and mathematical anomalies; while it remains an open problem, we fully expect 2D-VS modelling to play a key role in resolving some of the issues highlighted by Berry [6]. To this end, we have also taken some important steps forward in the understanding of eigenmode fractal dimension [9] in kaleidoscope-laser contexts. Preliminary calculations (undertaken with specialist fractal analysis software [10]) suggest that the power-spectrum dimension may be independent of transverse aperture symmetry.

4. References

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