Refraction and Goos-Hänchen Shifts of Spatial Solitons at Cubic-Quintic Interfaces

E. A. McCoy, J. M. Christian, and G. S. McDonald
Materials & Physics Research Centre, University of Salford, Greater Manchester M5 4WT, U.K.

J. Sánchez-Curto and P. Chamorro-Posada
ETSI Telecomunicación, Universidad de Valladolid, Campus Miguel Delibes Paseo Belén, E-47011 Valladolid, Spain
Author e-mail address: e.mccoy1@edu.salford.ac.uk

Abstract: We present the first analysis of spatial solitons refracting at the planar interface between dissimilar materials with both $\chi^{(3)}$ and $\chi^{(5)}$ optical susceptibilities. A nonparaxial Snell's law is derived and giant Goos-Hänchen shifts are predicted.

OCIS codes: (190.6135) Spatial solitons; (190.4350) Nonlinear optics at surfaces; (160.4330) Nonlinear optical materials

1. Wave-interface problems

In their most general form, wave-interface problems are inherently angular in nature. For instance, the interaction between light waves and material boundaries essentially defines the entire field of optics. The seminal works of Aceves *et al.* [1,2] considered scalar bright spatial solitons impinging on the planar interface between two dissimilar Kerr media. While these classic nonlinear-Schrödinger models undeniably paved the way toward understanding how self-collimated light beams behave at material discontinuities, they suffer from a fundamental limitation: the assumption of slowly-varying envelopes means that, in the laboratory frame, angles of incidence, reflection and refraction (relative to the interface) must be near-negligibly small. The intrinsic angular restriction may be eliminated by adopting a mathematical and computational framework that is based on the solution of nonlinear Helmholtz equations. To date, we have considered bright [3] and dark [4] soliton refraction in dissimilar focusing and defocusing Kerr materials, respectively.

Here, the first detailed account will be given of bright soliton refraction beyond the Kerr approximation. We consider the interface between media whose nonlinear polarization has contributions from both $\chi^{(3)}$ (Kerr) and $\chi^{(5)}$ susceptibilities [5]. Our model is based on an inhomogeneous Helmholtz equation with a cubic-quintic nonlinearity, and analysis is facilitated by knowledge of the exact solitons of the corresponding homogeneous problem [6].

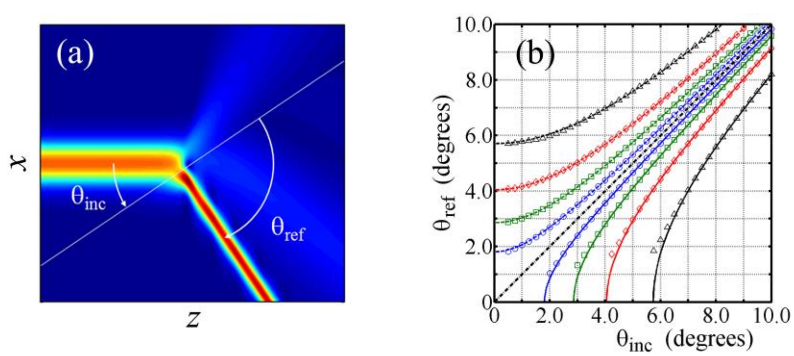

Fig. 1. (a) External refraction is characterized by $\theta_{\rm ref} > \theta_{\rm inc}$ (which becomes possible when the beam crosses into a denser medium). (b) Computational testing of the cubic-quintic Snell's law at a linear interface (Solid lines: theory. Points: numerics).

2. Nonparaxial refraction & Snell's law

By respecting field continuity conditions at the interface, a universal Snell's law may be derived for describing the refractive properties of soliton beams. If the angles of incidence and refraction in the laboratory frame are denoted by $\theta_{\rm inc}$ and $\theta_{\rm ref}$, respectively [see Fig. 1(a)], then

$$\gamma n_{01}\cos\theta_{\rm inc} = n_{02}\cos\theta_{\rm ref}, \quad \text{where} \quad \gamma \equiv \left[\frac{1+2\kappa\rho_0\left(1+\frac{2}{3}\sigma\rho_0\right)}{1+2\kappa\rho_0\left(\alpha+\frac{2}{3}\nu\sigma\rho_0\right)\left(1-\Delta\right)^{-1}}\right]^{1/2}, \tag{1}$$

and (n_{01}, n_{02}) are the linear refractive indexes, respectively. This compact law contains a multiplicative factor γ that captures the interplay between system nonlinearity (via the beam's peak intensity ρ_0 and the quintic coefficient σ), discontinuities in linear / nonlinear material properties (parametrized by Δ / α and ν), and finite beam waists [$\kappa <<$ $O(1)$ for broad scalar beams]. External refraction scenarios (where $\theta_{\mathrm{ref}} > \theta_{\mathrm{inc}}$ so that the soliton bends *away from* the interface) are inherently nonparaxial and have no counterpart in conventional (Schrödinger-type) frameworks [1,2]. Extensive computations have tested analytical predictions, providing strong supporting evidence for the validity of the Helmholtz modelling approach across wide regions of a six-dimensional parameter space [see Fig. 1(b)]. The Snell's law provides a simple algebraic result for the critical angle θ_{crit}, namely

$$\tan\theta_{\mathrm{crit}} \equiv \left\{ \frac{\Delta + 2\kappa\rho_0\left[\left(1-\alpha\right)+\frac{2}{3}\sigma\rho_0\left(1-\nu\right)\right]}{1-\Delta+2\kappa\rho_0\left(\alpha+\frac{2}{3}\nu\sigma\rho_0\right)} \right\}^{1/2}, \tag{2}$$

that is in good agreement with full simulations of solitons at linear and weakly-nonlinear interfaces.

3. Goos-Hänchen shifts

In the current context, the Goos-Hänchen (GH) shift [1,7] is a phenomenon whereby a reflecting beam experiences a translation in its outgoing trajectory (along the interface) relative to the path predicted by geometrical optics (i.e., plane wave theory) [see Figs. 2(a) and 2(b)]. These shifts are most pronounced close to the critical angle, and they can be greatly enhanced (often termed *giant*) in systems where the host media are nonlinear. Recently, we quantified the GH-shift characteristics of Helmholtz bright solitons at interfaces involving the Kerr nonlinearity [8], and particular attention was paid to regimes involving external linear refraction. Here, we will give an overview of similar considerations in material regimes where the $\chi^{(5)}$ susceptibility can no longer be neglected. Detailed numerical calculations, guided by Eq. (2), have uncovered new qualitative behaviour at highly nonlinear interfaces and we have uncovered shifts that are orders-of-magnitude greater than those in earlier studies [see Fig. 2(c)].

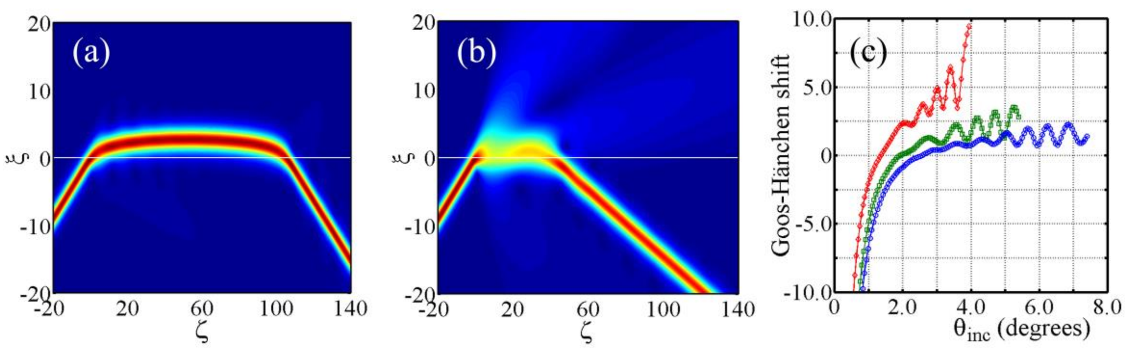

Fig. 2. Simulations (in normalized units) of typical GH shifts with external refraction at (a) linear and (b) nonlinear interfaces. (c) Numerical calculations of GH shifts (measured in units of diffraction length) in systems where linear refraction is external. Here, the $\chi^{(3)}$ susceptibility decreases across the interface (in the domain $\xi > 0$) while $\chi^{(5)}$ is uniform throughout.

4. References

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