The spatiotemporal Ginzburg-Landau equation: dissipative solitons & stability

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The complex Ginzburg-Landau (GL) equation describes universal wave propagation in dispersive systems that also exhibit competition between amplification and dissipation [1,2]. The balance between dispersive effects (group-velocity dispersion and self-phase modulation), linear gain and nonlinear loss can, in principle, lead to the formation of a stationary wavepacket (or *soliton*) in the local time frame. Here, we propose a novel two-fold generalization of the traditional GL equation to accommodate additional physical effects: (i) spatiotemporal dispersion [3], and (ii) power-law nonlinearity [4]. Exact analytical bright solitons of the new model have been derived, with asymptotic analysis demonstrating the emergence of well-known solutions [1,2] in a simultaneous multiple limit. Extensive simulations have revealed that, like its conventional counterpart (see Fig. 1), the new class of spatiotemporal dissipative soliton is also susceptible to a blow-up phenomenon (where the zero-amplitude continuous-wave solution is modulationally unstable against background fluctuations of arbitrarily-small magnitude). However, a route to stabilization may be possible by coupling the soliton to a non-dispersing linear wave [5].

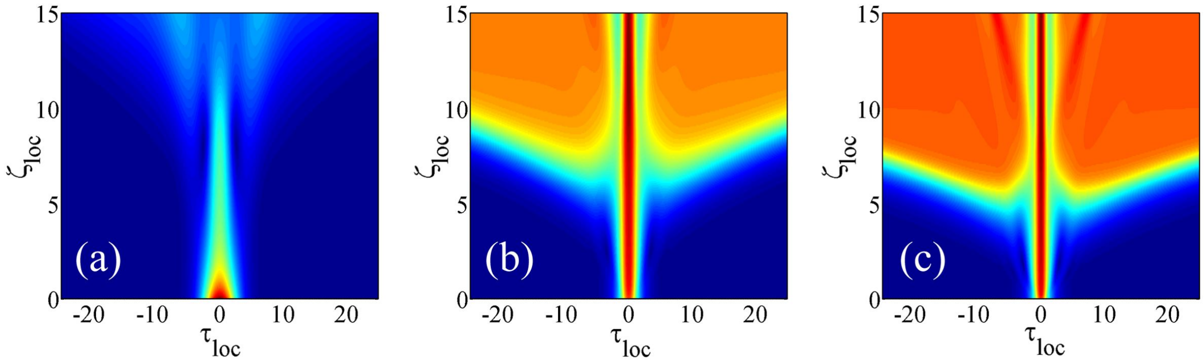

Figure 1. Instability in the conventional complex GL equation in the local time frame (τ_{loc}, ζ_{loc}) [2] for increasing strength of power-law nonlinearity. The perturbed initial-value problem corresponds to launching an input pulse whose peak intensity is lower than that predicted by the stationary solution for (a) sub-Kerr, (b) Kerr, and (c) super-Kerr systems.

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