Helmholtz solitons at nonlinear interfaces

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Reflection and refraction of spatial solitons at dielectric interfaces, accommodating *arbitrarily* angles of incidence, is studied. Analysis is based on Helmholtz soliton theory, which eliminates the angular restriction associated with the paraxial approximation. A novel generalization of Snell's law is discovered that is valid for collimated light beams and the entire angular domain. Our new theoretical predictions are shown to be in excellent agreement with full numerical simulations. New qualitative features of soliton refraction and limitations of previous paraxial analyses are highlighted. © 2007 Optical Society of America

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Interfaces play a fundamental role as boundary conditions for linear and nonlinear waves. In nonlinear systems, wave disturbances often evolve as localized states called solitons. Within photonics, the self-stabilizing character of such states led to the proposal of spatial optical solitons as elementary data bits in the storage and processing of optical information.[1,2] Soliton properties have been extensively studied by using the nonlinear Schrödinger equation, but this type of model has physical limitations. For example, it is valid only for broad beams of moderate intensity, whose propagation angles (relative to the reference axis) are negligibly small. Helmholtz soliton theory provides a framework for analyzing the propagation[3] and interaction[4] of broad, moderately intense beams at *arbitrary* angles with respect to the reference direction. This type of nonparaxiality differs, both physically and mathematically, from contexts involving strong self-focusing of highly intense, or ultranarrow beams.[5,6]

In this Letter, we study an elementary consideration in optics, namely, the properties of nonlinear beams at interfaces defined by two dissimilar dielectric media. This is an inherently angular problem, for which previous (paraxial) analyses are restricted to the consideration of vanishingly small angles of incidence.[7–9] Nonparaxial considerations are included only in those studies that, based on the split-field method,[10] analyze other types of nonlinear discontinuities, such as linear–nonlinear interfaces[11] and photorefractive crystals.[12] In this work, Helmholtz soliton theory is used to derive a model for Kerr-like media that allows description of arbitrary incidence angles. A remarkably compact generalization of Snell's law is uncovered. Analytical predictions for the refraction of nonlinear beams and the critical angle for reflection are verified by full numerical simulations.

We consider two focusing Kerr media separated by a planar boundary, as shown in Fig. 1. Each medium has a total refractive index described by $n_{0j}+\alpha_j I$, where $j=1,2$, α_j is the Kerr coefficient and I is light intensity. The angles of incidence and refraction are denoted θ_i and θ_t, respectively. Kerr coefficients are assumed to be sufficiently small that $n_j^2 \approx n_{0j}^2 + 2n_{0j}\alpha_j I$. In the scalar Maxwell field equation (Ref. 13), the forward-propagating complex electric field is written as $E(x,z)=E_0 u(x,z)\exp(ikz)$, and, without further approximation, a nonlinear Helmholtz equation for such inhomogeneous media is derived,

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i\frac{\partial u}{\partial \zeta} + \frac{1}{2}\frac{\partial^2 u}{\partial \xi^2} + |u|^2 u = \left[\frac{\Delta}{4\kappa} + (1-\alpha)|u|^2\right] H(\xi) u. \tag{1}$$

Equation (1) is thus completely equivalent to the corresponding Maxwell field equation. $H(\xi)$ is a Heaviside function (equal to zero for $\xi<0$, and unity for $\xi>0$), $\zeta=z/L_D$, $\xi=2^{1/2}x/w_0$, and w_0 is a transverse scale parameter, equal to the waist of a reference Gaussian beam of diffraction length $L_D=kw_0^2/2$. $\kappa=1/(kw_0)^2$ is the nonparaxiality parameter, k is spa-

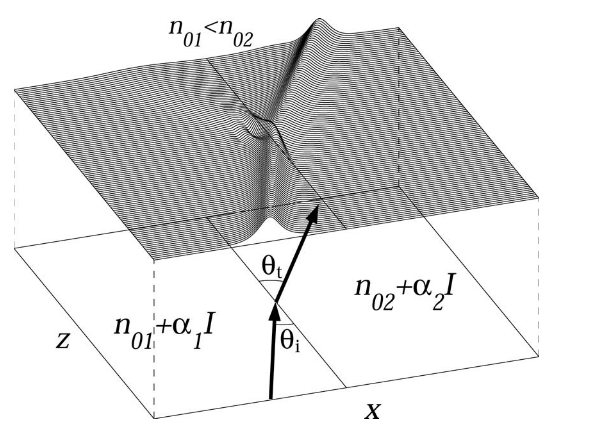

Fig. 1. Geometry of the interface problem: a spatial soliton is incident on a planar boundary (located at $x=0$).

tial wavenumber, and $E_0=(n_{01}/k\alpha_1 L_D n_{02})^{1/2}$. The parameters

$$\Delta \equiv 1-(n_{02}/n_{01})^2, \qquad \alpha \equiv \alpha_2/\alpha_1, \tag{2}$$

define the relative refractive properties of the adjoining media.

In each medium, Eq. (1) has an exact analytical bright soliton solution.[3,13] Field matching at the ($\xi=0$) interface is enforced through continuity of the phase of the refracting beam. We find that, in the second medium, the soliton is

$$u(\xi,\zeta)=\eta_0 \operatorname{sech}\left[\eta_0\sqrt{\alpha}\frac{\xi+V'\zeta}{\sqrt{1+2\kappa V'^2}}\right] \times \exp\left[i\sqrt{\frac{1+2\kappa\eta_0^2}{1+2\kappa V^2}}\left(-V'\xi+\frac{\zeta}{2\kappa}\right)\right] \times \exp\left(\frac{-i\zeta}{2\kappa}\right), \tag{3}$$

where

$$V'=\left\{V^2-\frac{1}{2\kappa}\frac{1+2\kappa V^2}{1+2\kappa\eta_0^2}[\Delta+(1-\alpha)2\kappa\eta_0^2]\right\}^{1/2} \tag{4}$$

is the beam's transverse velocity, which is related to the amplitude η_0 and velocity V of the incident beam. In the absence of a discontinuity, $\Delta \to 0$ and $\alpha \to 1$. This gives $V' \to V$, and one recovers the exact soliton solution in the first medium from Eqs. (3) and (4).

Equations (3) and (4) quantify three distinct types of nonparaxial correction.[13–15] When the sole source of nonparaxiality arises from oblique propagation (i.e., it is of Helmholtz type), one has $\kappa \ll 1$ (broad beams) and $\kappa{\eta_0}^2 \ll 1$ (moderate intensities), but κV^2 can become arbitrarily large. This follows from the geometrical identity[13] $\tan\theta=(2\kappa)^{1/2}V$, linking propagation angle θ in unscaled coordinates to velocity V and beam width (through κ).

The relations $\tan\theta_i=(2\kappa)^{1/2}V_i$ and $\tan\theta_t=(2\kappa)^{1/2}V'$ can be used with Eq. (4) to derive an, astonishingly simple, Helmholtz generalization of Snell's law that governs Kerr soliton refraction:

$$\gamma n_{01}\cos\theta_i = n_{02}\cos\theta_t, \tag{5}$$

where

$$\gamma \equiv \left[\frac{1+2\kappa\eta_0^2}{1+2\kappa\eta_0^2\alpha(1-\Delta)^{-1}}\right]^{1/2} \tag{6}$$

is a correction factor that depends only on the nonlinear term $\kappa\eta_0^2$ and the interface parameters Δ and α.

We stress that our attention, in the following analysis, is restricted to cases where $\kappa \ll 1$ and $\eta_0 \sim 1$, so that $\kappa\eta_0^2 \ll 1$ is always satisfied. We also consider $\alpha \sim 1$, so that there is only a moderate mismatch in the nonlinear refractive index. Under these conditions, the linear mismatch must be small so that the linear and nonlinear interface contributions can be comparable. When $|\Delta| \gg |1-\alpha|2\kappa\eta_0^2$, linear interface effects dominate, and Eq. (4) reveals a regime where beam refraction is effectively independent of intensity. Reflection and refraction characteristics of self-collimated nonlinear beams are then essentially indistinguishable from those of linear plane waves. When $|\Delta| \sim |1-\alpha|2\kappa\eta_0^2$, linear and nonlinear interface contributions are comparable, and beam reflection–refraction characteristics show a strong intensity dependence (thus deviating from predictions of linear plane-wave theory).

Analytical predictions are tested by comparison with numerical solutions[16] of Eq. (1). The characteristics at a purely linear interface ($\alpha=1$) are investigated first. As the beam crosses the interface ($\Delta=0.005$), its propagation angle is expected to decrease. Figure 2(a) demonstrates excellent agreement between the theoretical predictions of Eq. (5) (solid line) and full numerical simulations (points). For $\eta_0=1$ solitons and two values of κ (differing by more than a factor of 10), the nonlinear beam behavior is verified to converge to the linear plane-wave characteristic.

Discontinuity in the nonlinear refractive index is now considered. We choose here $\alpha=2$ and retain $\Delta=0.005$. For very broad $\eta_0=1$ solitons (for example, $\kappa{\eta_0}^2=10^{-4}$), the inequality $\Delta \gg |1-\alpha|2\kappa\eta_0^2$ is satisfied and, as predicted by Eq. (5), beam refraction characteristics converge to those of plane waves [see Fig. 2(b)]. On the other hand, for $\kappa{\eta_0}^2=2.5\times10^{-3}$, both linear and nonlinear interface effects are of the same order: $\Delta \sim |1-\alpha|2\kappa\eta_0^2$. Parameters were chosen to yield the interface transparency condition of $\theta_t=\theta_i$ and to ensure that there is only one soliton in the second medium, avoiding multiple soliton decomposition obtained for higher values of α. The agreement between theory and simulations, illustrated in both graphs of Fig. 2, also extends to larger values of angles of incidence. Figures 2(a) and 2(b) describe the propagation of the soliton transmitted in the second medium, although reflected power can also be found in the first medium (see Fig. 1). Numerical simulations show that the energy flow is always preserved at the interface.

One can define a critical angle of incidence θ_C, below which there is no refracted soliton in the second medium. Setting $V'=0$, or $\theta_t=0$, gives $\gamma(1-\Delta)^{-1/2}\cos(\theta_C)=1$, and

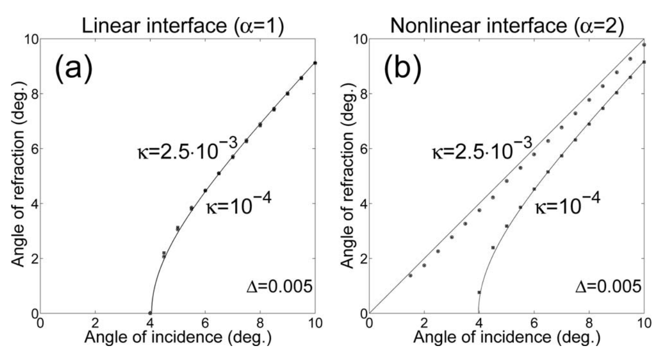

Fig. 2. Generalized Snell's law for Helmholtz solitons incident on a linear (left) and nonlinear (right) interface: theory (solid curves) and full simulations (dots).

$$\tan(\theta_C) = \left[\frac{\Delta + 2\kappa\eta_0^2(1-\alpha)}{1 - \Delta + 2\kappa\eta_0^2\alpha} \right]^{1/2}. \tag{7}$$

For linear plane waves impinging on a linear interface, the conventional Snell's law[7] states that the existence of a critical angle requires $n_{02}/n_{01}<1$. For collimated nonlinear beams at a linear interface (where $\alpha=1$), the corresponding condition is $0<\Delta<1$. Consequently, any interface satisfying this criterion possesses a critical angle. However, our model shows that interfaces with $\Delta<0$ may also have a critical angle provided that the linear mismatch is offset by a combination of nonlinear and Helmholtz effects, i.e., provided that $\Delta+2\kappa\eta_0^2(1-\alpha)>0$. In our simulations, we find that nonlinear surface waves are excited in the vicinity of the critical angle [Eq. (7)].[8]

Finally, analytical predictions for the critical angle are tested. The theoretical and numerical critical angles (solid curves and dots, respectively) are compared in Fig. 3, for $\alpha=2$ and $\eta_0=2$. Overall, our analytical predictions for nonlinear beam refraction prove very reliable. Critical angles for linear interfaces, as predicted by nonparaxial equation (7) and the corresponding paraxial model,[7] are also shown in Fig. 3. The paraxial model is found to yield valid results only under the restrictive condition $\Delta \to 0$ (i.e., when the linear discontinuity becomes extremely small).

In conclusion, we have studied the behavior of broad, moderately intense spatial soliton beams at interfaces for a wide range of incidence angles. A nonlinear Snell's law is reported that captures the full angular nature of the problem studied; numerical simulations were employed, and they confirm our analytical predictions. New qualitative soliton refraction effects were also uncovered, since the generality of our model permits the description of interface geometries with $\Delta<0$. Large quantitative corrections to existing paraxial models of nonlinear beam refraction have been demonstrated. Other associated effects, such as the nonlinear Goos–Hänchen shift, nonlinear surface waves, and soliton decomposition into multiple beams, are reserved for a forthcoming study.

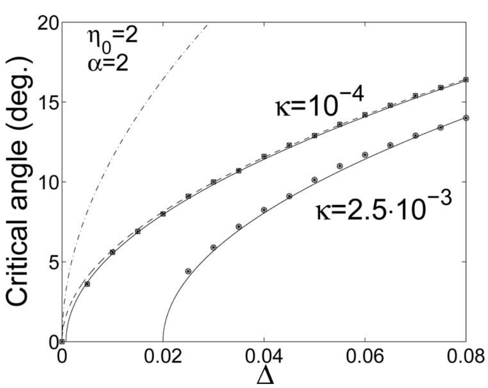

Fig. 3. Critical angle for Helmholtz solitons: theory (solid curves) and numerics (dots).

The refraction of beams at interfaces is a fundamental consideration, and we expect that this work will have important implications in the design of integrated-optic devices. The universality of solitons in nonlinear systems, as well as the importance of material boundaries, also suggests that applications may be found in other research fields.

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