CAPACITORS AND DIELECTRICS

Important Definitions and Units

Capacitance:

The property of a system of electrical conductors and insulators which enables it to store electric charge when a potential difference exists between the conductors.

Units: farad (Coulomb per volt)

Defined as the capacitance of a capacitor between the plates of which there appears a potential drop of 1 volt when it is charged with one coulomb of electricity

Dielectric:

A non-conductor of electricity, insulator. A substance in which an electric field gives rise to no net flow of electric charge but only to a displacement of charge.

Dielectric constant or relative permittivity:

The ratio of the capacitance of a capacitor with the given substance as the dielectric, to the capacitance of the same capacitor with air (or a vacuum) as the dielectric. Symbol κ.

Dielectric strength:

The maximum voltage which can be applied to a dielectric material without causing it to break down. Usually expressed in volts per mm.

Capacitors

Spheres

The potential of a positively charged conducting sphere is given by:

$$V_+' = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Where q is the charge on the sphere of radius R.

If another sphere of the same radius R, carrying a negative charge -q, is located at a distance >>R from the first sphere then it can be said that both spheres are electrically isolated. The potential of the second sphere is therefore given by:

$$V_-' = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

The potential difference between the two spheres is thus:

$$V' = V_+' - V_-' = \frac{1}{4\pi\varepsilon_0}\frac{2q}{R}$$

The potential difference is therefore proportional to the charge on either sphere.

This equation can now be written:

$$q = (2\pi\varepsilon_0 R)V' = C'V'$$

C', the proportionality constant, is called the capacitance of the two spheres.

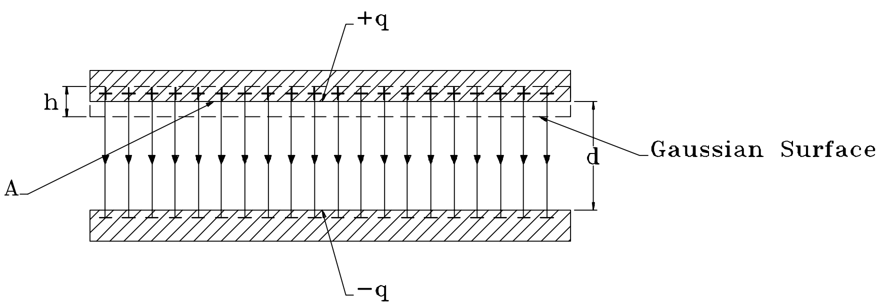
When the two spheres are moved closer together then our initial equations, for the potential of a sphere, do not hold as they were derived with the assumption that the field around each possessed spherical symmetry - this can no longer be the case as the lines of force which emanated uniformly from the sphere, are now affected by the presence of the second sphere.

When a positive charge is brought close to an object then it has the effect of raising that objects potential. When a negative charge is brought close to an object then it has the effect of lowering that objects potential. Thus, when the two spheres are brought closer together, the positive sphere will have the effect of raising the potential of the negative sphere from V_-' to V_- and the negative sphere will lower the potential of the positive sphere from V_+' to V_+.

It is therefore simple to deduce that, although the charge on each sphere has remained constant, the potential difference between the spheres has been reduced and the capacitance has been increased.

$$q = CV \qquad \text{where } C>C' \text{ and } V<V'$$

Parallel plate capacitor



This capacitor is formed from two plates with area A separated by a distance d. When the plates are connected to the terminals of a battery then a positive charge +q will appear on one plate and a negative charge -q will appear on the other. If we make the assumption that d is small compared to the area of the plates then it can be said that a uniform field E exists between the plates, with the lines of force being parallel and evenly spaced.

Using Gauss's law, it is possible to calculate the capacitance of the device. The Gaussian surface is of height h with an area equal to that of the plates (shown on the diagram). The only part of the Gaussian surface we are concerned with is that which lies between the plates as the surface which lies within the plate has zero field and the two side surfaces are at 90° to the field. For the surface between the plate E is constant and therefore the flux is equal to EA

Therefore: $\varepsilon_0\phi = \varepsilon_0 EA = q$

The work needed to take a test charge q_0 from one plate to the other is equal to q_0V or the force (q_0E) multiplied by the distance (d). These expressions must be equal i.e.

$$q_0V = q_0Ed$$

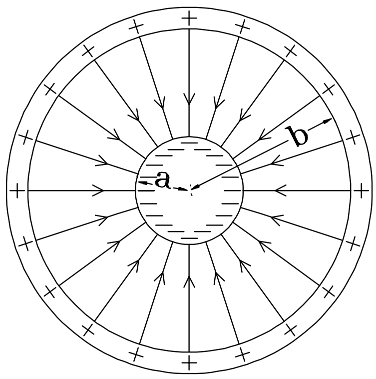
$$V = Ed$$

Substituting into the standard capacitor equation C=q/V we get:

$$C = \frac{q}{V} = \frac{\varepsilon_0 EA}{Ed} = \frac{\varepsilon_0 A}{d}$$ Parallel plate capacitor

A cylindrical capacitor

A cylinder capacitor is made up of two coaxial cylinders of radius a and b and of length l.



We assume that the length l >> b and construct a Gaussian surface which is a cylinder of radius r and length l, where b > r > a

Using Gauss's law:

$$\frac{q}{\varepsilon_0} = \oint E.dA$$

$$q = E\varepsilon_0 2\pi rl$$

$$E = \frac{q}{2\pi rl\varepsilon_0}$$

The potential difference between the plates is:

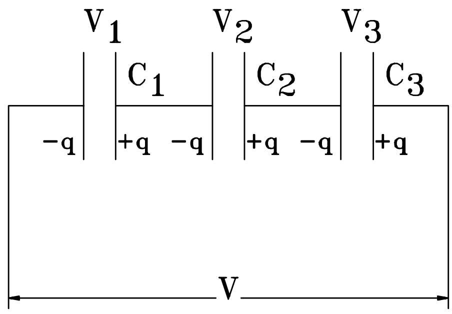
$$V = \int_a^b E dr = \int_a^b \frac{q}{2\pi\varepsilon_0 l}\frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 l}\ln\frac{b}{a}$$

The capacitance is given by:

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

The capacitance only depends on the geometry of the capacitor. If we look back at the equation for a parallel plate capacitor it can be seen that the capacitance is also only a function of the dimensions of the device.

Capacitors in Series



For capacitors connected in series, the magnitude of the charge q on each of the plates must be equal. This is due to the fact that charge is conserved and the net charge must therefore be zero for each pair of plates (whether they be from the same capacitor or different capacitors).

Applying the standard equation q = CV to each capacitor:

$V_1 = q/C_1 \qquad V_2 = q/C_2 \qquad V_3 = q/C_3$

The total potential for the series combination is:

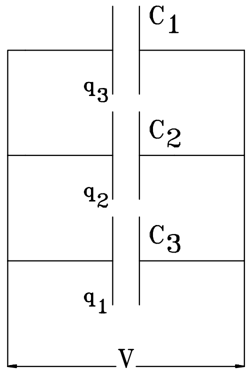
$V = V_1 + V_2 + V_3$
$V = q/C_1 + q/C_2 + q/C_3 = q(1/C_1 + 1/C_2 + 1/C_3)$

The equivalent capacitance is therefore:

$C = q/V = 1/(1/C_1 + 1/C_2 + 1/C_3)$

$\mathbf{1/C = 1/C_1 + 1/C_2 + 1/C_3}$

Capacitors in Parallel



When a set of capacitors are connected in parallel then it follows that the voltage across each is constant. The charge on each capacitor will differ and is given by q = CV. The total net charge, before a battery is connected is zero, this is still the case after the battery is connected but now the charge has separated with each capacitor having a net charge of zero.

The total charge q will be:

$$q = q_1 + q_2 + q_3$$
$$q = C_1V + C_2V + C_3V$$
$$q = (C_1 + C_2 + C_3)\ V$$

The equivalent capacitance C is:

$$C = q/V = (C_1 + C_2 + C_3)\ V/V$$

$$\mathbf{C = C_1 + C_2 + C_3}$$