Black and grey soliton refraction at interfaces

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The behaviour of grey solitons incident on Kerr defocusing interfaces is studied within the framework of Helmholtz theory. We report that the refraction of grey solitons is governed by a generalized Snell's law whose validity extends to arbitrary angles. Unlike their bright or black counterparts, grey solitons may exhibit either external or internal refraction depending solely on the soliton greyness parameter.

Spatial soliton refraction at interfaces separating two nonlinear media has been traditionally studied in terms of the paraxial Nonlinear Schrödinger (NLS) equation, which limits the validity of results to vanishingly small angles of incidence [1]. This restriction is removed in a Helmholtz nonparaxial framework [2,3], in which a Nonlinear Helmholtz (NLH) equation [4] describes the evolution of a broad beam (when compared to the wavelength) propagating at arbitrary angles.

Nonlinear Helmholtz (NLH) Equation

$$\kappa\frac{\partial^2 u}{\partial\zeta^2}+i\frac{\partial u}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial\xi^2}\pm|u|^2u=\left(\frac{\Delta}{4\kappa}\pm(1-\alpha)|u|^2\right)H(\xi)u$$

Interface parameters

$$\Delta\equiv\frac{n_{01}^2-n_{02}^2}{n_{01}^2},\quad \alpha\equiv\frac{|\alpha_2|}{|\alpha_1|}$$

Nonparaxiality parameter

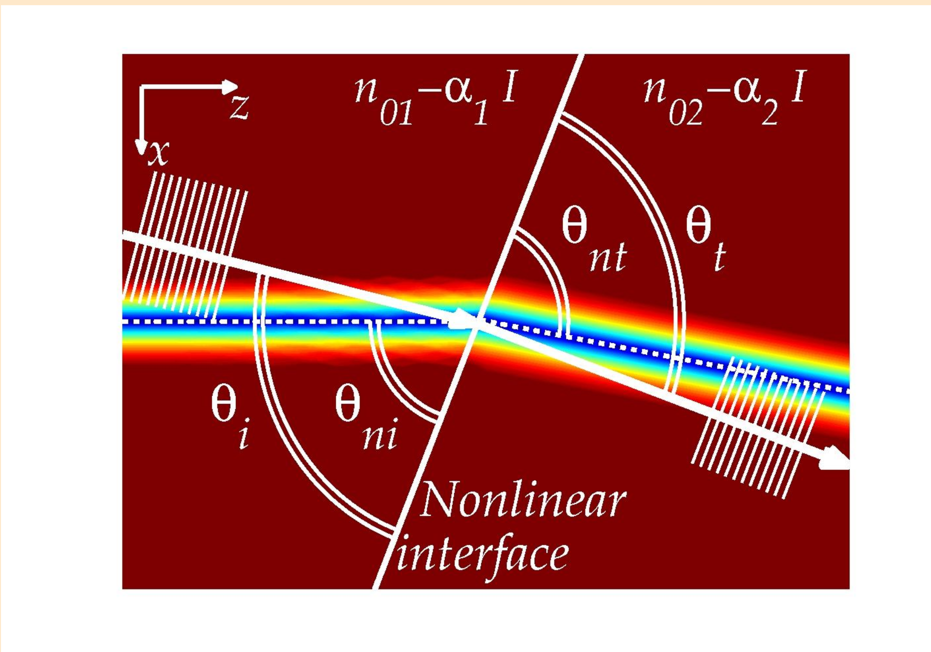
$$\kappa=\frac{1}{k^2w_0^2}$$

Generalized Snell's law for bright, black and grey solitons

We find that the full angular content inherent in the behaviour of solitons at nonlinear interfaces is described by a generalized Snell's law which is valid for both bright and dark soliton refraction [5,6].

$$\gamma_\pm n_{01}\cos(\theta_{ni}+\theta_{0i})=n_{02}\cos(\theta_{nt}+\theta_{0t})$$

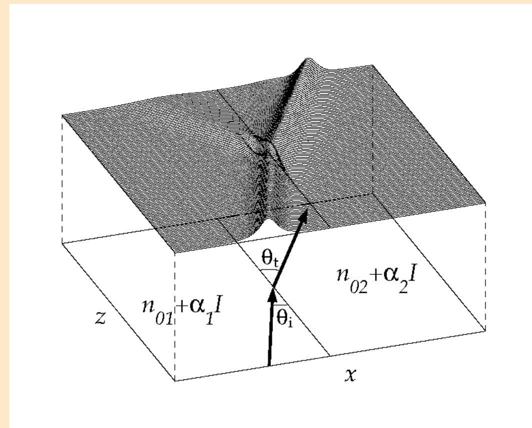
1. θ_{ni} and θ_{nt} are the net angles of incidence and refraction of a soliton, respectively. They account for the total angle between the propagation direction of the soliton hump (bright) or dip (dark) and the nonlinear interface.
2. $\theta_{0i}=\theta_i-\theta_{ni}$ and $\theta_{0t}=\theta_t-\theta_{nt}$ represent the intrinsic angles of the incident and refracted grey soliton relative to the propagation direction of the background wave (white arrow) supporting the corresponding dark soliton, respectively. For bright and black solitons, $\theta_{0i}=\theta_{0t}=0$.
3. $\gamma_\pm$ is a nonlinear correction where ± corresponds to either Kerr focusing (+) or defocusing (-) media.



Bright solitons

$$\gamma_+\equiv\left[\frac{1+2\kappa\eta_0^2}{1+2\kappa\eta_0^2(1-\Delta)^{-1}}\right]^{1/2}\qquad \theta_{0i}=\theta_{0t}=0$$

η_0 is the bright soliton amplitude

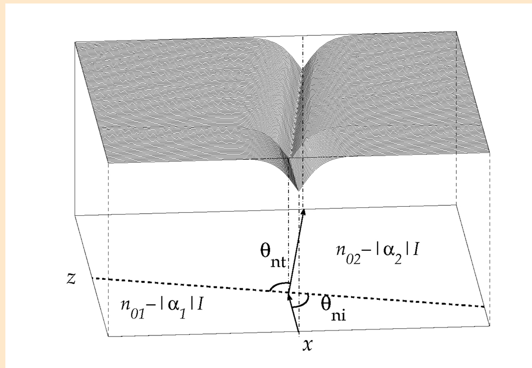


Dark solitons

$$\gamma_-\equiv\left[\frac{1-4\kappa u_0^2}{1-4\kappa u_0^2\alpha(1-\Delta)^{-1}}\right]^{1/2}$$

$$\tan(\theta_{0i})=\sqrt{2\kappa}\frac{u_0F}{\left(1-(2+F^2)2\kappa u_0^2\right)^{1/2}}$$

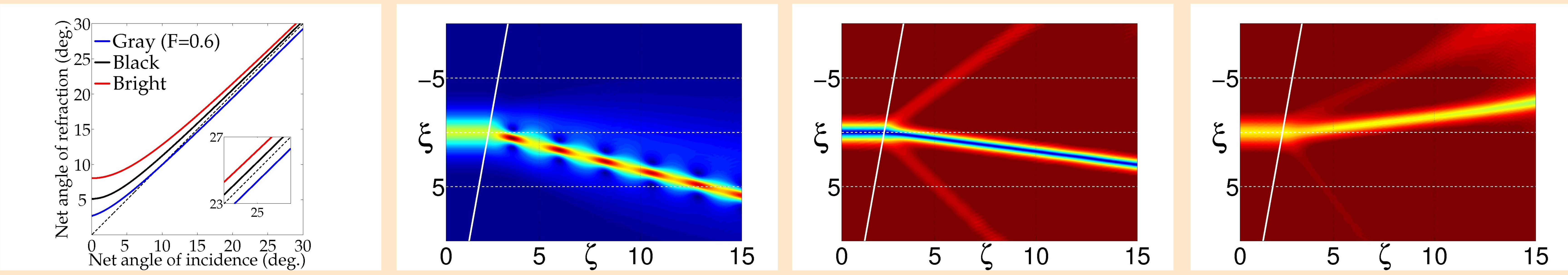
$$\tan(\theta_{0t})=\sqrt{2\kappa}\frac{u_0F\alpha^{1/2}}{\left(1-\Delta-(2+F^2)2\kappa u_0^2\alpha\right)^{1/2}}$$



u_0 is the amplitude of the background beam supporting the soliton and F is the soliton greyness parameter

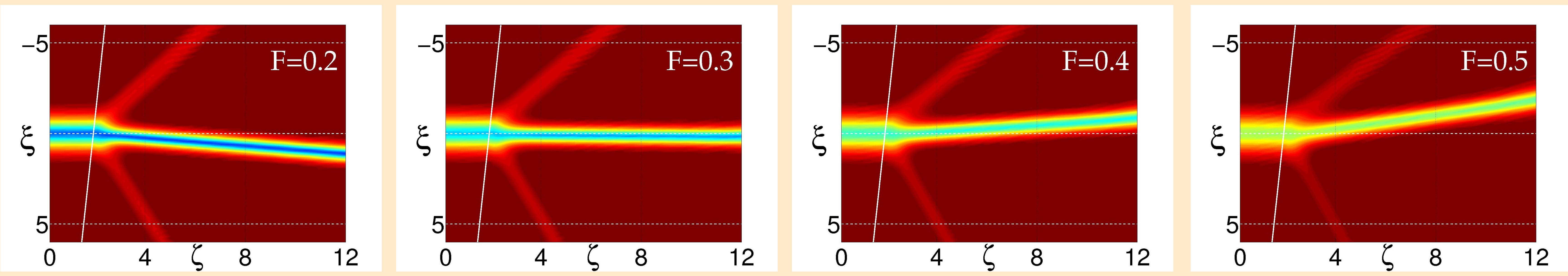
Theoretical results predicted by the generalized Snell's law are displayed in the left plot of the figure below for a nonlinear interface with $n_{02}=1.008n_{01}$ and $\alpha_2=3\alpha_1$. The inset shows that for certain angles of incidence bright and black solitons undergo external refraction, while internal refraction is experienced by a grey soliton. The three snapshots correspond to numerical results obtained for a bright, black ($F=0$) and grey ($F=0.6$) soliton impinging on the same nonlinear interface at 25°. There is excellent agreement between theoretical predictions and results from full numerical simulations.

Generalized Snell's law | Bright (external refraction) | Black (external refraction) | Grey (internal refraction)



Grey soliton refraction dependence on the greyness parameter

Grey soliton refraction is predicted to be strongly dependent on the greyness parameter F. This effect is verified in the snapshots below, which show numerical results of grey solitons impinging on a nonlinear interface ($n_{02}=1.0124n_{01}$ and $\alpha_2=4\alpha_1$) at 30°. Soliton greyness parameters are increasing from F=0.2 to F=0.5, in intervals of 0.1. The set of figures reveals that the net angle of refraction θ_{nt} decreases as F grows, resulting in a transition of grey soliton refraction from external to internal regimes. This behaviour is completely governed by the generalized Snell's law. Larger values of F increase the angular component associated with the intrinsic grey excitation of the refracted soliton θ_{0t} [7], thus reducing the net angle of refraction θ_{nt}.



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