Propagation of Helmholtz Solitons at Nonlinear Interfaces

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Abstract: The reflection and refraction properties of soliton beams at nonlinear interfaces and arbitrary angles is analyzed using the nonlinear Helmholtz equation. The results highlight limitations of previous studies based on the paraxial nonlinear Schrödinger equation.

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1. Introduction

The behaviour of spatial solitons at nonlinear interfaces has been analysed both analytically and numerically [1]. However, previous studies have been based on the nonlinear Schrödinger equation (NSE) and their validity is restricted to the paraxial limit and, thus, vanishingly small angles of incidence.

In recent years, the use of the nonlinear Helmholtz equation (NHE) has provided a more general non-paraxial theory [2,3] which permits the study of soliton propagation at arbitrary angles. In this work, a generalized nonlinear Helmholtz equation (NHE) for studying the behaviour of Helmholtz solitons at the interface of two Kerr focusing media is presented and the behaviour of the Helmholtz soliton solutions is analysed by using analytical considerations. The results are supported by simulations using well-tested numerical methods [4].

2. Generalized NHE and Helmholtz solitons.

We consider the propagation of soliton beams in an inhomogeneous 2D space where two distinct focusing Kerr media are separated by an interface at x=0 as described in Figure 1.

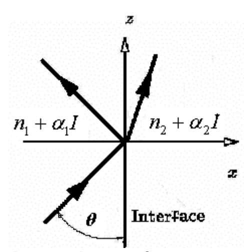

Fig.1. Coordinate axes used in the analysis. The interface separating the two Kerr media is found at x=0

In Figure 1, θ is the angle of incidence of a bright soliton at the interface, $n_i + \alpha_i I$, i =1,2, is the total refractive index of the medium i and I is the optical intensity. The complex envelope A of a CW optical field $E(x,z)=A(x,z)\exp(ikz)$ evolves according to the generalized nonlinear Helmholtz equation

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i\frac{\partial u}{\partial \zeta} + \frac{1}{2}\frac{\partial^2 u}{\partial \xi^2} + |u|^2 u = \left(\frac{\Delta}{4\kappa} + (1-\alpha^{-1})|u|^2\right) H(\xi) u \qquad (1)$$

with the normalizations [2,3]

$$\zeta = \frac{z}{L_D} \quad \xi = \frac{\sqrt{2}x}{w_0} \quad u(\xi,\zeta) = \sqrt{\frac{k\alpha_1 L_D n_2}{n_1^2}} A(\xi,\zeta) \quad L_D = \frac{kw_0^2}{2} \quad \Delta = \frac{n_1^2 - n_2^2}{n_1^2} \quad \alpha = \frac{\alpha_1}{\alpha_2}, \qquad (2)$$

where $H(\xi)$ is the Heaviside function and $\kappa=1/(kw_0)^2$ is a non-paraxiality parameter [2]. The relations between the linear and nonlinear contributions to the refractive index at both sides of the discontinuity are given by Δ and α, respectively.

Eq. 1 is a generalisation for the spatially inhomogeneous case of a nonparaxial nonlinear Schrödinger (NNLS) equation which is fully equivalent to the NHE [2]. The exact soliton solution in the second medium reads

$$u(\xi,\zeta)=\eta_2 \operatorname{sec}h\left(\frac{\eta_2'(\xi+V_2\zeta)}{\sqrt{1+2\kappa V_2^2}}\right)\exp\left[i\frac{\sqrt{1-\Delta+2\kappa\eta_2'^2}}{\sqrt{1+2\kappa V_2^2}}\left(-V_2\xi+\frac{\zeta}{2\kappa}\right)\right]\exp\left(-i\frac{\zeta}{2\kappa}\right) \tag{3}$$

where

$$\eta_2'=\eta_2\sqrt{\alpha^{-1}}. \tag{4}$$

When α=1 and Δ=0, Eq. 3 gives the soliton solution for the first medium [2] by making the substitutions $\eta_2\rightarrow\eta_1$ and $V_2\rightarrow V_1$. The paraxial soliton solution is recovered in the limit $\kappa\rightarrow 0$, $\kappa V^2\rightarrow 0$ and $\kappa\eta^2\rightarrow 0$ [2]. Since $2\kappa V^2=\tan^2\theta$ [2], where θ is the propagation angle in the unscaled reference frame, the second condition forces vanishingly small propagation angles for the paraxial approximation to be valid.

3. Discontinuities in the linear refractive index.

When the nonlinear response is continuous across the interface, α^{-1}=1, the soliton refraction angle is determined by the effect of the discontinuity in the linear refractive index. The continuity of the phase of the solitons at ξ=0, assuming that $\kappa\eta^2$<<1, gives the condition

$$V_2=\sqrt{V_1^2(1-\Delta)-\frac{\Delta}{2\kappa}}. \tag{5}$$

By using the expression $\tan^2(\theta)=2\kappa V^2$, Eq. 5 can be written in terms of the propagation angles in the unscaled reference frame as Snell's law of refraction, $n_1\cos(\theta_1)=n_2\cos(\theta_2)$. This result is confirmed by the numerical results obtained from the integration of (1), as shown in Fig. 2.

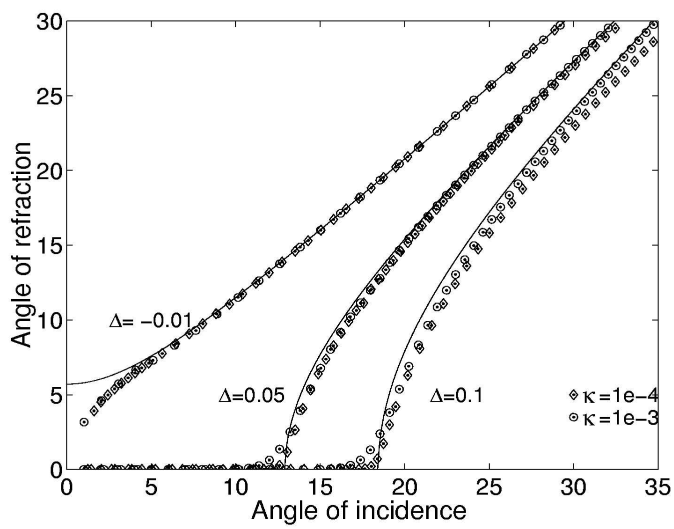

Fig.2. Refraction angle for different values of κ and Δ: numerical results (points) and values from Eq. 5 (solid lines).

Whereas paraxial analyses [1] are restricted to $\Delta>0$, solitons propagating at wide angles resulting from the refraction when $\Delta<0$ can also be studied in the NHE framework.

Figure 3 shows the power reflection coefficient in various situations. For $\Delta>0$, the condition for total internal reflection can be written in terms of soliton parameters as $V<Vc$, where $Vc=(2\kappa(1-\Delta)/\Delta)^{-1/2}$. In the paraxial case, the reflection coefficient for linear propagation of optical beams coincides with Fresnel's formula (dashed line) [1] since the finite beam width is neglected. In the full Helmholtz analysis, the results for linear beam propagation (points) diverge from the result for plane waves as κ is increased from zero. The numerical results show how the reflection coefficient for the solitons (lines-points) deviates from the predicted linear result including the effect of the finite beam width (points) as the incident soliton becomes narrower (larger κ), showing a transition from a quasi-linear behaviour for small κ to a highly nonlinear regime as $\Delta/4\kappa$ approaches 1.

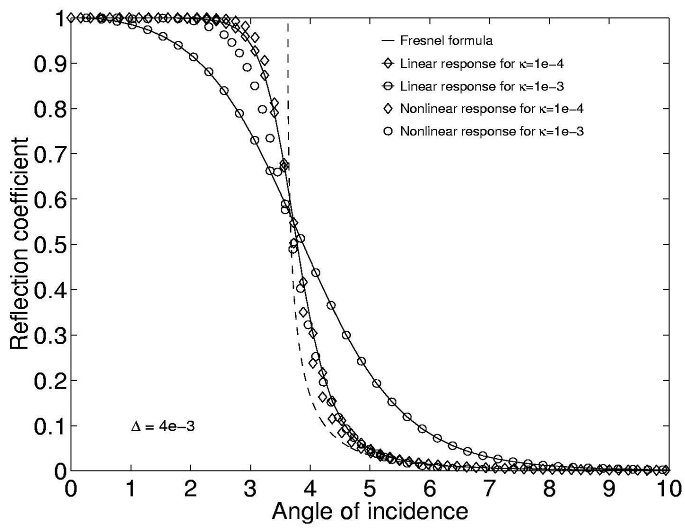

Fig.3. Reflection coefficient showing the transition from the quasi-linear behaviour to the highly nonlinear regime as κ changes ($\Delta=4.10^{-3}$).

4. Discontinuities in the nonlinear refractive index.

When the linear part of the refractive index is continuous across the interface, $\Delta=0$, the soliton in the second medium preserves the propagation direction but changes substantially its width depending on $\alpha^{-1/2}$. As the nonlinear refractive index of a medium increases ($\alpha^{-1}>1$), the soliton becomes narrower and its associated power decreases in relation to $\alpha^{-1/2}$. The opposite effect is obtained when $\alpha^{-1}<1$.

Therefore, when a soliton propagates in a nonlinear medium and crosses into a second one with $\alpha_1<\alpha_2$, the soliton splits into a series of narrower beams due to its excess power. The number and amplitudes of the resulting solitons depend on $\alpha^{-1.}$ On the other hand, when $\alpha^{-1}<1$, the power of the incident soliton may not be high enough to create a soliton in the second medium, thus leading to a progressive diffractive broadening of the resulting beam in the second medium.

The numerical integration of (1) in the case $\alpha^{-1}>1$ evidences one important difference with the paraxial theory [1], which establishes that the number of solitons depends uniquely on the strength of the nonlinearity. In the Helmholtz framework, the soliton pattern created in the second medium is determined by both α and the soliton angle of incidence θ. Figure 4 shows how the number and trajectories of the resulting beams are modified when the angle of incidence is increased (from left to right) and the strength of the nonlinearity is fixed.

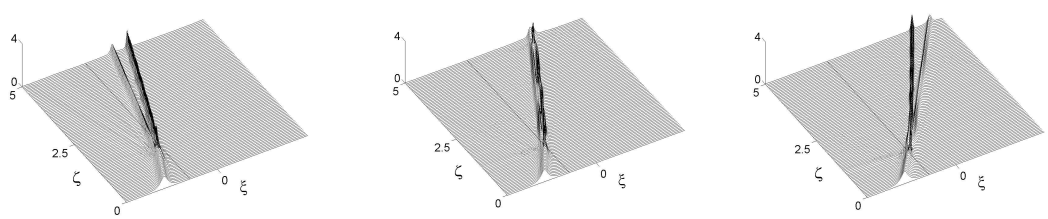

Fig.4. Formation of different soliton patterns when α^{-1} is fixed and the angle of incidence varies.

5. References

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