Reflection and refraction of Helmholtz solitons at nonlinear interfaces

J. Sánchez-Curto
P. Chamorro-Posada
Departamento de Teoría de la Señal y Comunicaciones e Ingeniería Telemática,Universidad de Valladolid, ETSI Telecomunicación, Campus Miguel Delibes s/n, 47011 Valladolid, Spain

G.S. McDonald
Joule Physics Laboratory, School of Computing, Science and Engineering, University of Salford, Salford M5 4WT, UK

The behaviour of Helmholtz solitons at the interface between two nonlinear media is analysed. The use of the full nonlinear Helmholtz equation (NHE) permits to address soliton reflections and refractions at arbitrary angles. The results highlight the limitation of previous studies based on the paraxial nonlinear Schrödinger equation.

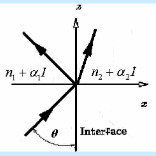
Introduction

The reflection and refraction properties of optical solitons incident at interfaces between two focusing Kerr-type media have been previously studied using the nonlinear Schrödinger equation (NSE) [1]. The validity of such results is restricted to vanishingly small angles of incidence because of the use of the paraxial approximation.

In this work, the nonlinear Helmholtz equation (NHE) [2,3]

$$\kappa\frac{\partial^2 u}{\partial\zeta^2}+i\frac{\partial u}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial\xi^2}+|u|^2u=\left(\frac{\Delta}{4\kappa}+(1-\alpha^{-1})|u|^2\right)H(\xi)u$$

is used in order to obtain a description of the evolution of bright optical solitons in Kerr focusing media valid for arbitrary angles.



$$\zeta=\frac{z}{L_D}\quad \xi=\frac{\sqrt{2}x}{w_0}\quad u(\xi,\zeta)=\sqrt{\frac{k\alpha_1 L_D n_2}{n_1^2}}A(\xi,\zeta)\quad and\quad L_D=\frac{kw_0^2}{2}$$

$$\Delta=\frac{n_1^2-n_2^2}{n_1^2},\quad \alpha=\frac{\alpha_1}{\alpha_2}$$

$$\kappa=\frac{1}{k^2w_0^2}$$

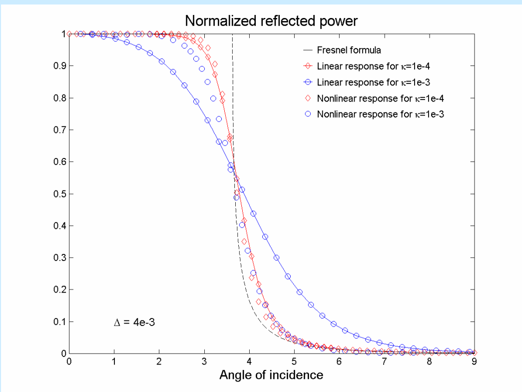
Discontinuities in the linear refractive index

A) Reflection coefficient

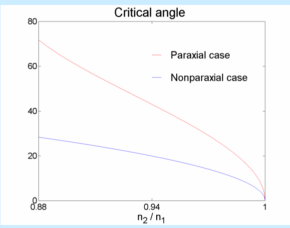
By setting $\alpha^{-1}=1$, the analysis is restricted to discontinuities in the linear refractive index. The figure below shows the ratio of total reflected power to incident power:

- The *linear response* is the result that would be obtained for an optical beam corresponding to the soliton profile in the absence of nonlinearity in either media. In [1], such result is coincident with the *Fresnel formula* since the finite extent of the beam angular spectrum is neglected in the paraxial limit.
- The *nonlinar response* shows the actual results obtained from the numerical solution of the NHE [4].

For very broad soliton beams ($\kappa=10^{-4}$), there is no significative difference between the linear and nonlinear responses. As $\Delta/4\kappa=O(1)$ one enters a highly nonlinear regime with significative deviations from the linear response.



The figure besides compares the actual value of the critical angle to that of the paraxial aproximation [1]. They validity of the paraxial results is restricted to $\Delta\to 0$.



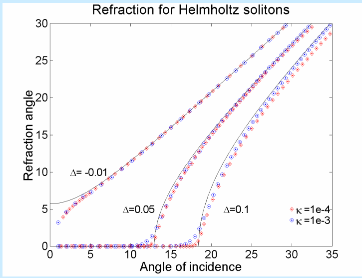
B) Refraction angle

Snell law, $n_1\cos(\theta_1)=n_2\cos(\theta_2)$, which defines the refraction angle for linear plane waves, can be rewritten in the normalizations used as

$$V_2=\sqrt{V_1^2(1-\Delta)-\frac{\Delta}{2\kappa}}$$

where V_1 and V_2 are the transverse velocities associated to the actual propagation angle as $\tan^2\theta=2\kappa V^2$.

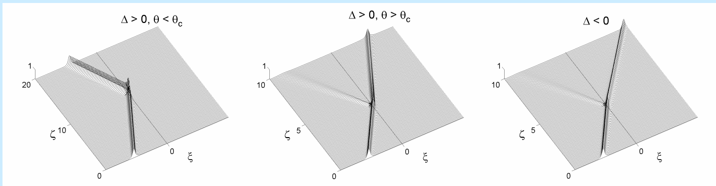
For $\Delta>0$, the condition $V_2=0$ sets a real value for V_1 which defines the critical transverse velocity for total internal reflection.



The refraction angle is obtained by the condition that wavefronts match at the discontinuity. Optical solitons are perfectly collimated beams and their behaviour is very close to that predicted from the Snell law when the nonlinear response of the medium is continuous across the interface.
The figure compares the results from the numerical simulations (points) to the predictions of Snell law of refraction.

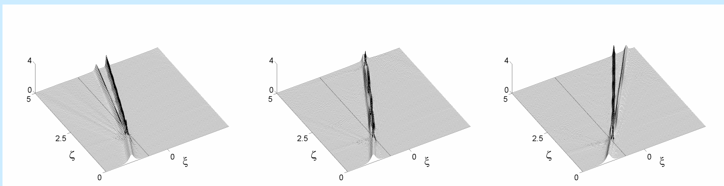
Whereas paraxial analyses are limited to $\Delta>0$, solitons propagating at wide angles resulting from the refraction when $\Delta<0$ can also be analyzed in the NHE framework.

The most significative soliton evolutions for different values of Δ and θ are shown below.



Discontinuities in the nonlinear refractive index

When $\alpha^{-1}>1$, a group of solitons is created in the second medium. The number of such solitons depends on the strength of the nonlinearity α_2. Whereas the paraxial results exhibit a null dependence with the propagation angle [1], the full Helmholtz analysis reveals that the angle of incidence plays an important role in order to fix the strength and number of the resulting solitons. In the figure below α is fixed while θ is increasing (from left to right), showing that a proper combination of α and θ cancels the formation of a multisoliton scheme.



References

1. A. B. Aceves, J. V. Moloney and A. C. Newell, "Theory of light-beam propagation at nonlinear interfaces.", Phys. Rev. A, 39, 4 (1989).
2. P. Chamorro-Posada, G. S. McDonald and G. H. C. New, "Exact soliton solutions of the nonlinear Helmholtz equation: communication," *J. Opt. Soc. Am. B* 19, 1216 (2002).
3. P. Chamorro-Posada, G. S. McDonald and G. H. C. New, "Propagation properties of nonparaxial spatial solitons," *J. Mod. Opt.* 47, 1877 (2000).
4. P. Chamorro-Posada, G. S. McDonald and G. H. C. New, "Nonparaxial beam propagation methods," *Opt. Commun.* 19, 1 (2001).