PULSES WITH RELATIVISTIC AND PSEUDO-RELATIVISTIC ASPIRATIONS

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Abstract

Temporal solitons are robust self-localizing pulses that play a pivotal role in modern understandings of nonlinear wave phenomena. Here, we propose a new approach for describing pulses in dispersive nonlinear systems. By retaining the fuller generality of the underlying Helmholtz formulation, one may go beyond conventional modelling (which anticipates slowly-varying field envelopes) to capture novel and potentially exploitable physical effects. While specific attention is paid to guided-wave optics, we believe these results are universal in nature and can be applied in a wide range of contexts, such as plasmas and water waves.

Several distinct classes of exact solution family (continuous waves, solitons, cnoidal waves) are derived. The stability of these solutions is then investigated with analytical and semi-analytical methods, and one can sometimes deploy inverse-scattering theory. The fully second-order governing equation may be integrated using fast computational techniques, and extensive numerical analysis is used to test theoretical predictions.

Conventional theory of pulses

It can be safely said that optical soliton pulses are one of the most thoroughly investigated phenomena in nonlinear photonics. Since the seminal works of Hasegawa and Tappert [1,2], the cornerstone of many investigations has been the slowly-varying envelope approximation (SVEA). In combination with a subsequent Galilean boost to a local time frame, the SVEA provides a mathematical device commonly used to reduce the complexity of the longitudinal (spatial) part of wave operator. While this approach has some clear-cut advantages [e.g. by replacing the elliptic (or hyperbolic) governing equation with a parabolic one], there are some physical effects that fall outside its remit. One such effect is spatial dispersion, recently discussed by Biancalana and Creatore in the context of pulse envelope equations in semiconductor planar waveguides [3].

Conventional pulse theory has enjoyed unbridled longevity in the literature over the past forty years for two main reasons. Firstly, it often provide an adequate description of the phenomena being observed. Secondly, a large body of knowledge exists on how to solve the resultant parabolic governing equations.

Helmholtz theory of pulses

Here, we report on our new Helmholtz approach to nonlinear pulse modelling, whereby the classic "SVEA + Galilean boost" is omitted. Mathematical [4] and computational [5] methods are deployed that are similar to these used over the past 12 years to analyze nonlinear beams. Our results have a simple physical interpretation, and some tantalizing connections to Einstein's special relativity theory have been uncovered.

We begin by considering a scalar electric field $E(t,z) = E_0 u(t,z)\exp[i(k_0 z - \omega_0 t)] + \text{c.c.}$ that is travelling down the longitudinal axis z of a Kerr waveguide, and where t is the time coordinate. Here, $u(t,z)$ is the dimensionless envelope that modulates a carrier wave with amplitude E_0, centre frequency ω_0 and propagation constant $k_0 = n_0\omega_0/c$, where n_0 is the linear refractive index of the core medium at ω_0 and c is the vacuum speed of light. The transverse spatial variation of the electric field is controlled by the structure of the waveguide itself. Using standard Fourier decomposition techniques, one can show that the normalized wave envelope satisfies [6]

$$\kappa\frac{\partial^2 u}{\partial \zeta^2} + i\left(\frac{\partial u}{\partial \zeta} + \alpha\frac{\partial u}{\partial \tau}\right) + \frac{s}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0. \quad (1)$$

The space and time coordinates are $\zeta = z/L$ and $\tau = t/t_p$, respectively, where t_p is the duration of a reference pulse and $L = t_p^2/|k_2|$, where k_2 is the group-velocity dispertion (GVD) coefficient. The sign of GVD is flagged by $s = \pm 1 = \text{sgn}(-k_2)$ (+1 for anomalous; -1 for normal) and $\alpha \equiv k_1 t_p/|k_2|$, where $1/k_1$ is the group speed. The small parameter $\kappa = \kappa_0 + D$ encapsulates two contributions: $\kappa_0 \equiv 1/2k_0 L > 0$ is inherent to any electromagnetic mode, while D (which may assume either sign) is a medium contribution from spatial dispersion [3]. This latter effect appears in some semiconductors, and its origin lies in field-exciton coupling.

When considering pulse propagation problems, one typically follows a well-prescribed route to go from the nonlinear Helmholtz equation (1) to the more straightforward nonlinear Schrödinger model. Firstly, one typically

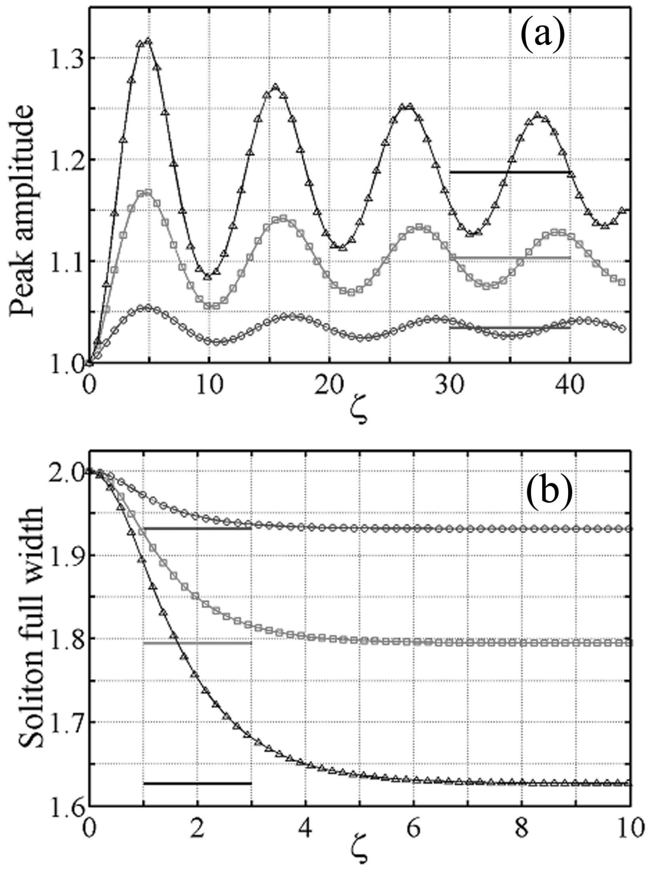

Figure 1: Self-reshaping of (a) bright and (b) dark solitons under small, medium, and large perturbations to the local pulse shape. Horizontal bars denote predictions of inverse-scattering theory [7].

invokes the SVEA by arguing that $|\kappa \partial^2 u/\partial\zeta^2|$ is small. A Galilean boost to a frame moving at the group speed $1/\alpha$ is then implemented by defining coordinates $\tau_{\text{loc}} = \tau - \alpha\zeta$ and $\zeta_{\text{loc}} = \zeta$.

The precedent for using models such as Eq. (1) was set more than 30 years ago [6], but the approach seems to have received little subsequent attention. A huge amount of progress can actually be made with Eq. (1), which contains just one extra term compared to the spatial Helmholtz equation [4,5]. Full convergence to conventional pulse theory is uncovered in a simultaneous multiple limit. One of the key results and also one of the most interesting is the velocity combination rule for Helmholtz solitons. This law, which is geometric in nature and *independent of system nonlinearity*, is strongly reminiscent of the way velocities add together in relativistic kinematics. In fact, when $\text{sgn}(s\kappa) = +1$, there is a one-to-one mapping with special relativity theory. Deeper insight can be gained by considering the transformation laws for Eq. (1), which show that the velocity combination rule is an intrinsic property of the model itself, rather than a property of particular (e.g., soliton) solutions.

Solitons and stability

We will give an overview of our investigations into Eq. (1) and related models with more general nonlinearities (e.g., cubic-quintic and saturable). Exact analytical bright and dark solitons will be reported, and their space-time geometry considered in detail. New parameter regimes will be considered that have no counterpart in the spatial domain namely, $\kappa < 0$ (in the spatial domain, κ must remain positive [4,5]), and a wide range of generic features will also be identified. Crucially, the properties of Helmholtz temporal solitons are found to depend on the sign of the product $s\kappa$, rather than sgn(s) or sgn(κ) separately. Extensive computations also examine their role as robust attractors in the system dynamics.

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