

## 1. Introduction

In the package **Introduction to Matrices** the basic rules of *addition* and *subtraction* of matrices, as well as *scalar multiplication*, were introduced. The rule for the *multiplication of two matrices* is the subject of this package. The first example is the simplest.

Recall that if  $M$  is a matrix then the transpose of  $M$ , written  $M^T$ , is the matrix obtained from  $M$  by writing the rows of  $M$  as the columns of  $M^T$ .

If  $A = (a_1 \ a_2 \ \dots \ a_n)$  is a  $1 \times n$  (row) matrix and  $B = (b_1 \ b_2 \ \dots \ b_n)^T$  is a  $n \times 1$  (column) matrix then the product  $AB$  is defined as

$$AB = (a_1 \ a_2 \ \dots \ a_n) \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

This general rule is sometimes called the *inner product*.

**N.B.** The *row matrix* is on the left and the *column matrix* is on the right.

**Example 1** In each of the following cases, find the product  $AB$ .

(a)  $A = (1 \ 2)$ ,  $B = (4 \ 3)^T$ .      (b)  $A = (1 \ 1 \ 1)$ ,  $B = (2 \ 3 \ 4)^T$ .

(c)  $A = (1 \ -1 \ 2 \ 3)$ ,  $B = (1 \ 1 \ -3 \ 2)^T$ .

**Solution**

(a)  $AB = (1 \ 2) \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 1 \times 4 + 2 \times 3 = 4 + 6 = 10$ .

(b)  $AB = (1 \ 1 \ 1) \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 1 \times 2 + 1 \times 3 + 1 \times 4 = 2 + 3 + 4 = 9$ .

(c)  $AB = (1 \ -1 \ 2 \ 3) \begin{pmatrix} 1 \\ 1 \\ -3 \\ 2 \end{pmatrix} = 1 \times 1 + 1 \times (-1) + 2 \times (-3) + 3 \times 2$   
 $= 1 + (-1) + (-6) + 6 = 0$ .

## Matrix Multiplication

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The aim of this document is to provide a short, self assessment programme for students who wish to learn how to multiply matrices.

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## Table of Contents

1. Introduction
  2. Matrix Multiplication 1
  3. Matrix Multiplication 2
  4. The Identity Matrix
  5. Quiz on Matrix Multiplication
- Solutions to Exercises  
Solutions to Quizzes

The full range of these packages and some instructions, should they be required, can be obtained from our web page [Mathematics Support Materials](#).

**Solution**

$$(a) AB = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \times 4 + 2 \times 3 \\ 3 \times 4 + (-1) \times 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 9 \end{pmatrix}$$

$$(b) AB = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & -3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \times 2 + 1 \times 3 + 1 \times 4 \\ (-2) \times 2 + 1 \times 3 + (-3) \times 4 \end{pmatrix} = \begin{pmatrix} 9 \\ -13 \end{pmatrix}$$

The following observations on  $AB$  are worth noting.

- The element in the *first row* of  $AB$  is the *inner product* of the *first row* of  $A$  with the column matrix  $B$ .
- The element in the *second row* of  $AB$  is the *inner product* of the *second row* of  $A$  with the column matrix  $B$ .
- The number of *columns* of  $A$  must be equal to the number of *rows* of  $B$ .
- If  $A$  is  $2 \times n$  and  $B$  is  $n \times 1$  then  $AB$  is  $2 \times 1$ .

This rule for multiplication may be extended to matrices,  $A$ , which have more than two rows. For example, if  $A$  had 3 rows then the resulting matrix,  $AB$ , would have a third row; the value of this element would be the *inner product* of the *third row* of  $A$  with the column matrix  $B$ .

**EXERCISE 2.** For each of the cases below, calculate  $AB$ . (Click on the green letters for solutions.)

$$(a) A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}, \quad B = (4 \ 3)^T.$$

$$(b) A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix}, \quad B = (2 \ 3 \ 4)^T.$$

$$(c) A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}, \quad B = (4 \ 3)^T.$$

$$(d) A = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix}, \quad B = (5 \ -4 \ 3 \ 2)^T.$$

**EXERCISE 1.** For each of the cases below, calculate  $AB$ . (Click on the green letters for solutions.)

$$(a) A = (-2 \ 4), \quad B = (3 \ 2)^T,$$

$$(b) A = (5 \ 3 \ -2), \quad B = (3 \ -4 \ 2)^T,$$

$$(c) A = (4 \ 4 \ -2 \ -3), \quad B = (5 \ -4 \ 3 \ 2)^T.$$

The following observations are worth noting.

- The row matrix is on the left, the column matrix is on the right.
- The *row* and *column* have the same number of elements.
- The inner product  $AB$  is a  $1 \times 1$  matrix, i.e. a *number*.
- Nothing has yet been said about a matrix product  $BA$ .

**Quiz** If  $A = (x \ x \ 1)$  and  $B = (x \ 6 \ 9)^T$ , which of the following values of  $x$  will result in  $AB = 0$ ?

$$(a) x = 1, \quad (b) x = 3, \quad (c) x = -3, \quad (d) x = -2.$$

## 2. Matrix Multiplication 1

The previous section gave the rule for the multiplication of a row vector  $A$  with a column vector  $B$ , the *inner product*  $AB$ . This section will extend this idea to more general matrices.

Suppose that  $A = \begin{pmatrix} a_1 & a_2 & \dots & a_n \\ c_1 & c_2 & \dots & c_n \end{pmatrix}$  and  $B = (b_1 \ b_2 \ \dots \ b_n)^T$ .

Then

$$AB = \begin{pmatrix} a_1 & a_2 & \dots & a_n \\ c_1 & c_2 & \dots & c_n \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a_1 b_1 + a_2 b_2 + \dots + a_n b_n \\ c_1 b_1 + c_2 b_2 + \dots + c_n b_n \end{pmatrix}$$

**Example 2** Find  $AB$  for each of the following cases.

$$(a) A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}, \quad B = (4 \ 3)^T.$$

$$(b) A = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & -3 \end{pmatrix}, \quad B = (2 \ 3 \ 4)^T.$$



- If  $A$  is  $m \times n$  and  $B$  is  $n \times r$  then the product  $AB$  exists.
- The resulting matrix is  $m \times r$ .  $((m \times n)(n \times r) = m \times r)$
- The element in the  $i$ th row,  $j$ th column of the matrix  $AB$  is the *inner product* of the  $i$ th row of  $A$  with the  $j$ th column of  $B$ .

**Example 3** Find the element in the *2nd* row *3rd* column of  $AB$  if

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 4 & -2 \\ 3 & -1 & 2 \end{pmatrix}.$$

**Solution** Since  $A$  is  $2 \times 2$  and  $B$  is  $2 \times 3$ , the product  $AB$  exists and is a  $2 \times 3$  matrix. The required element is the inner product of the *second row* of  $A$  with the *third column* of  $B$ , i.e.

$$(-1) \times (-2) + 3 \times 2 = 2 + 6 = 8.$$

**EXERCISE 4.** If

$$A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$$

find the  $ij$  element, i.e. the element in the  $i$ th row  $j$ th column, of  $AB$  for the following cases. (Click on the green letters for solutions.)

- (a)  $i = 3, j = 2,$       (b)  $i = 2, j = 3,$       (c)  $i = 1, j = 2,$   
 (d)  $i = 2, j = 1,$       (e)  $i = 3, j = 1,$       (f)  $i = 1, j = 3,$

**Quiz** Which of the following is the element in the *3rd* row, *3rd* column, of the matrix  $AB$  in the above exercise?

- (a) 26,      (b) -26,      (c) -12,      (d) 12.

### 3. Matrix Multiplication 2

The extension of the concept of matrix multiplication to matrices,  $A, B$ , in which  $A$  has more than one row and  $B$  has more than one column is now possible. The product matrix  $AB$  will have the same number of columns as  $B$  and each column is obtained by taking the product of  $A$  with each column of  $B$ , in turn, as shown below.

Let  $A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$  and let  $b_1, b_2$  be the first and

second columns of  $B$  respectively. Then

$$Ab_1 = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 13 \\ 4 \end{pmatrix} \quad \text{and} \quad Ab_2 = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}.$$

Thus

$$AB = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 11 & -2 \\ 13 & 4 \\ 4 & 5 \end{pmatrix}.$$

**EXERCISE 3.** For each of the cases below, calculate  $AB$ . (Click on the green letters for solutions.)

(a)  $A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix},$        $B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}.$

(b)  $A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix},$        $B = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}.$

(c)  $A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix},$        $B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}.$

(d)  $A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix},$        $B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}.$

**NB** The rules for finding the product of two matrices are summarised on the next page.

**Example 5** The matrices  $A$ ,  $B$  are

$$A = \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix}.$$

Calculate  $AB$  and  $BA$ .

**Solution** Using the rules of matrix multiplication,

$$AB = \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I.$$

$$BA = \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I.$$

The matrix  $B$  is the *inverse* of the matrix  $A$ , and this is usually written as  $A^{-1}$ . Equally, the matrix  $A$  is the *inverse* of the matrix  $B$ . The equation  $AA^{-1} = A^{-1}A = I$  is always true.

## 5. Quiz on Matrix Multiplication

Let  $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -2 & 1 & 2 \end{pmatrix}$ ,  $C = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{pmatrix}$ .

Choose the correct option from the following.

Begin Quiz

- The  $2 \times 3$  element of  $AB$  is  
(a)  $-1$ , (b)  $1$ , (c)  $0$ , (d)  $2$ .
- The  $3 \times 1$  element of  $CB$  is  
(a)  $3$ , (b)  $-1$ , (c)  $4$ , (d)  $-6$ .
- The  $2 \times 2$  element of  $CA$  is  
(a)  $4$ , (b)  $-3$ , (c)  $0$ , (d)  $2$ .
- (a)  $B = C^{-1}$ , (b)  $A = B^{-1}$ , (c)  $C = A^{-1}$

End Quiz

Score:

Correct

## 4. The Identity Matrix

If  $A$  and  $B$  are two matrices, the product  $AB$  can be found if the number of *columns* of  $A$  equals the number of *rows* of  $B$ . If  $A$  is  $2 \times 3$  and  $B$  is  $3 \times 5$  then  $AB$  can be calculated but  $BA$  does not exist. The *order* in which matrices are multiplied together matters. Even when  $AB$  and  $BA$  both exist it is usually the case that  $AB \neq BA$ .

There is one particular matrix, the *identity matrix*, which has very special multiplication properties. The  $n \times n$  *identity matrix* is the  $n \times n$  matrix with 1s and 0s *as shown below*.

**Example 4** The  $2 \times 2$ ,  $3 \times 3$  and  $4 \times 4$  identity matrices are

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The most important property of the identity matrix is revealed in the following exercise.

**EXERCISE 5.** If the *identity matrix* is denoted by  $I$  and the matrix  $M$  is

$$M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix},$$

use the appropriate identity matrix to calculate the following matrix products. (Click on the *green* letters for solutions.)

- (a)  $IM$ , where  $I$  is the  $2 \times 2$  identity matrix, (b)  $MI$ , where  $I$  is the  $3 \times 3$  identity matrix.

In matrix multiplication the identity matrix,  $I$ , behaves exactly like the number 1 in ordinary multiplication. This was seen in the previous exercise. For part (a), the matrix  $I$  is the  $2 \times 2$  identity matrix; in part (b),  $I$  was  $3 \times 3$ ; they satisfy the equation  $IM = M = MI$ .



**Exercise 1(c)**

If the row matrix  $A = (4 \ 4 \ -2 \ -3)$  and the column matrix

$$B = \begin{pmatrix} 5 \\ -4 \\ 3 \\ -2 \end{pmatrix}$$

are multiplied, their **inner product**  $AB$  is

$$\begin{aligned} (4 \ 4 \ -2 \ -3) \begin{pmatrix} 5 \\ -4 \\ 3 \\ -2 \end{pmatrix} &= 4 \times 5 + 4 \times (-4) + (-2) \times 3 + (-3) \times (-2) \\ &= 20 - 16 - 6 + 6 = 4. \end{aligned}$$

Click on the green square to return

**Exercise 2(a)**

For the  $2 \times 2$  matrix

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$$

and the column  $B = (4 \ 3)^T$ , the product  $AB$  is

$$AB = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} (-2) \times 4 + 4 \times 3 \\ 5 \times 4 + 3 \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 29 \end{pmatrix}.$$

Click on the green square to return

**Solutions to Exercises****Exercise 1(a)**

If the row matrix  $A = (-2 \ 4)$  and the column matrix

$$B = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

are multiplied, the resulting **inner product** is

$$\begin{aligned} AB = (-2 \ 4) \begin{pmatrix} 3 \\ 2 \end{pmatrix} &= -2 \times 3 + 4 \times 2 \\ &= -6 + 8 \\ &= 2. \end{aligned}$$

Click on the green square to return

**Exercise 1(b)**

If the row matrix  $A = (5 \ 3 \ -2)$  and the column matrix

$$B = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$$

are multiplied, the resulting **inner product** is

$$\begin{aligned} AB = (5 \ 3 \ -2) \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} &= 5 \times 3 + 3 \times (-4) + (-2) \times 2 \\ &= 15 - 12 - 4 = -1. \end{aligned}$$

Click on the green square to return



**Exercise 2(d)**

If the  $2 \times 4$  matrix

$$A = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix}$$

is multiplied with the column matrix  $B = (5 \ -4 \ 3 \ 2)^T$ , the resulting product,  $AB$ , is

$$AB = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ -4 \\ 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \times 5 + 4 \times (-4) + (-2) \times 3 + (-3) \times 2 \\ 3 \times 5 + (-1) \times (-4) + (-1) \times 3 + 2 \times 2 \end{pmatrix} = \begin{pmatrix} -8 \\ 20 \end{pmatrix}.$$

Click on the green square to return

**Exercise 3(a)**

Let  $A$  and  $B$  be the  $2 \times 2$  matrices:

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}.$$

The matrix  $AB$  is

$$\begin{aligned} AB &= \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} -2 \times (-2) + 4 \times 5 & -2 \times 4 + 4 \times 3 \\ 5 \times (-2) + 3 \times 5 & 5 \times 4 + 3 \times 3 \end{pmatrix} \\ &= \begin{pmatrix} 24 & 4 \\ 5 & 29 \end{pmatrix}. \end{aligned}$$

Click on the green square to return

**Exercise 2(b)**

If the  $2 \times 3$  matrix

$$A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix}$$

and the column matrix  $B = (2 \ 3 \ 4)^T$  are multiplied together, then the resulting product  $AB$  is

$$\begin{aligned} AB &= \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \times 2 + 3 \times 3 + 2 \times 4 \\ 4 \times 2 + (-1) \times 3 + (-1) \times 4 \end{pmatrix} \\ &= \begin{pmatrix} 27 \\ 1 \end{pmatrix}. \end{aligned}$$

Click on the green square to return

**Exercise 2(c)**

If the  $3 \times 2$  matrix is

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}$$

and the column matrix is  $B = (4 \ 3)^T$ , then the product  $AB$  is

$$AB = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} (-2) \times 4 + 4 \times 3 \\ 5 \times 4 + 3 \times 3 \\ 4 \times 4 + (-1) \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 29 \\ 13 \end{pmatrix}$$

Click on the green square to return



**Exercise 3(d)**

If  $A$  is  $2 \times 3$  and  $B$  is  $3 \times 2$  given by the following

$$A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix},$$

then the matrix product  $AB$  is

$$\begin{aligned} AB &= \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 5 \times (-2) + 3 \times 5 + 2 \times 4 & 5 \times 4 + 3 \times 3 + 2 \times (-1) \\ 4 \times (-2) + (-1) \times 5 + (-1) \times 4 & 4 \times 4 + (-1) \times 3 + (-1) \times (-1) \end{pmatrix} \\ &= \begin{pmatrix} 13 & 27 \\ -17 & 14 \end{pmatrix}. \end{aligned}$$

Click on the green square to return □

**Exercise 4(a)**

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the  $(32)$  element in the matrix  $AB$ ,  $(AB)_{32}$ , is the inner product of the *third row* of  $A$  with the *second column* of  $B$ , i.e.

$$\begin{aligned} (AB)_{32} &= 2 \times 1 + 3 \times (-2) + (-1) \times (-3) + (-2) \times 0 \\ &= 2 - 6 + 3 + 0 = -1. \end{aligned}$$

Click on the green square to return □

**Exercise 3(b)**

If  $A$  and  $B$  are the  $2 \times 2$  matrices:

$$A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix},$$

then the matrix product  $AB$  is

$$\begin{aligned} AB &= \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 3 \times 5 + 2 \times (-7) & 3 \times (-2) + 2 \times 3 \\ 7 \times 5 + 5 \times (-7) & 7 \times (-2) + 5 \times 3 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

This is called the  $2 \times 2$  **identity matrix**.

Click on the green square to return □

**Exercise 3(c)**

If  $A$  and  $B$  are the matrices

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$$

then the matrix product  $AB$  is

$$\begin{aligned} AB &= \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} (-2) \times (-2) + 4 \times 5 & (-2) \times 4 + 4 \times 3 \\ 5 \times (-2) + 3 \times 5 & 5 \times 4 + 3 \times 3 \\ 4 \times (-2) + (-1) \times 5 & 4 \times 4 + (-1) \times 3 \end{pmatrix} \\ &= \begin{pmatrix} 24 & 4 \\ 5 & 29 \\ -13 & 13 \end{pmatrix}. \end{aligned}$$

Click on the green square to return □

## Exercise 4(d)

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the (21) element in the matrix  $AB$ ,  $(AB)_{21}$ , is the inner product of the *second row* of  $A$  with the *first column* of  $B$ , i.e.

$$\begin{aligned} (AB)_{21} &= 7 \times (-2) + (-8) \times 0 + (-6) \times 4 + 2 \times 0 \\ &= -14 + 0 - 24 + 0 = -38. \end{aligned}$$

Click on the green square to return



## Exercise 4(e)

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the (31) element in the matrix  $AB$ ,  $(AB)_{31}$ , is the inner product of the *third row* of  $A$  with the *first column* of  $B$ , i.e.

$$\begin{aligned} (AB)_{31} &= 2 \times (-2) + 3 \times 0 + (-1) \times 4 + (-2) \times 0 \\ &= -4 + 0 - 4 + 0 = -8. \end{aligned}$$

Click on the green square to return



## Exercise 4(b)

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the (23) element of  $AB$ ,  $(AB)_{23}$ , is the inner product of the *second row* of  $A$  with the *third column* of  $B$ , i.e.

$$\begin{aligned} (AB)_{23} &= 7 \times (-3) + (-8) \times (-5) + (-6) \times (-7) + 2 \times 6 \\ &= -21 + 40 + 42 + 12 = 73. \end{aligned}$$

Click on the green square to return



## Exercise 4(c)

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the (12) element in the matrix  $AB$ ,  $(AB)_{12}$ , is the inner product of the *first row* of  $A$  with the *second column* of  $B$ , i.e.

$$\begin{aligned} (AB)_{12} &= 1 \times 1 + (-2) \times (-2) + 4 \times (-3) + 5 \times 0 \\ &= 1 + 4 - 12 + 0 \\ &= -7. \end{aligned}$$

Click on the green square to return





**Exercise 5(b)**

For the  $2 \times 3$  matrix  $M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix}$ , the **right identity** matrix (multiplying  $M$  on the **right** to obtain  $MI$ ) is the  $3 \times 3$  matrix  $I$ :

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The product  $MI$  is thus

$$\begin{aligned} MI &= \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \times 1 + 2 \times 0 + 4 \times 0 & 1 \times 0 + 2 \times 1 + 4 \times 0 & 1 \times 0 + 2 \times 0 + 4 \times 1 \\ 7 \times 1 + 8 \times 0 + 6 \times 0 & 7 \times 0 + 8 \times 1 + 6 \times 0 & 7 \times 0 + 8 \times 0 + 6 \times 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} = M. \end{aligned}$$

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**Solutions to Quizzes****Solution to Quiz:**

Multiplying the row matrix  $A = (x \ x \ 1)$  with the column matrix

$$B = \begin{pmatrix} x \\ 6 \\ 9 \end{pmatrix}$$

from the **left** we have

$$\begin{aligned} AB &= (x \ x \ 1) \begin{pmatrix} x \\ 6 \\ 9 \end{pmatrix} = x \times x + x \times 6 + 1 \times 9 \\ &= x^2 + 6x + 9 = (x + 3)^2. \end{aligned}$$

Therefore the inner product  $AB = 0$ , if  $x = -3$ .

End Quiz

**Exercise 4(f)**

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the **(13)** element in the matrix  $AB$ ,  $(AB)_{13}$ , is the inner product of the **first row** of  $A$  with the **third column** of  $B$ , i.e.

$$\begin{aligned} (AB)_{13} &= 1 \times (-3) + (-2) \times (-5) + 4 \times (-7) + 5 \times 6 \\ &= -3 + 10 - 28 + 30 = 9. \end{aligned}$$

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**Exercise 5(a)**

$$\text{For the } 2 \times 3 \text{ matrix } M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix},$$

the **left identity** matrix (multiplying  $M$  on the **left** to obtain  $IM$ ) is the  $2 \times 2$  matrix  $I$ :

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The product  $IM$  is then

$$\begin{aligned} IM &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 1 \times 1 + 0 \times 7 & 1 \times 2 + 0 \times 8 & 1 \times 4 + 0 \times 6 \\ 0 \times 1 + 1 \times 7 & 0 \times 2 + 1 \times 8 & 0 \times 4 + 1 \times 6 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} = M. \end{aligned}$$

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**Solution to Quiz:**

The matrices  $A$  and  $B$  from Exercise 4 are

$$A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}.$$

The (33) element in the matrix of  $AB$ ,  $(AB)_{33}$ , is the inner product of the *third row* of  $A$  with the *third column* of  $B$ , i.e.

$$\begin{aligned} (AB)_{33} &= 2 \times (-3) + 3 \times (-5) + (-1) \times (-7) + (-2) \times 6 \\ &= -6 - 15 + 7 - 12 = -26. \end{aligned}$$

End Quiz